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Introduction to optimal auto-guiding: How to get the most from your set-up

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Deterministic setup tracking errors (open loop)

- Periodic error PE (unless direct drive):
  - Can be learned and partially corrected (PEC), high resolution encoders (on RA & DEC shafts)

- Polar alignment errors $\boldsymbol{\theta}$ and drift $\boldsymbol{\delta}$:
  - Minimized by good alignment ($\boldsymbol{\theta < 1' \rightarrow \delta_{max} \cong {}^{1}/_{3}\,''/min}$)
  - Limited by atmospheric refraction to about $\boldsymbol{\theta \cong 1'}$
  - Can be learned/predicated and partially corrected, sky model, auto-guiding.

- Flexure (OTA, mount, focuser/accessories, pier, guide-scope,…)
  - Minimized with a rigid setup and ONAG/OAG.
  - Can be learned & partially corrected, setup model, auto-guiding

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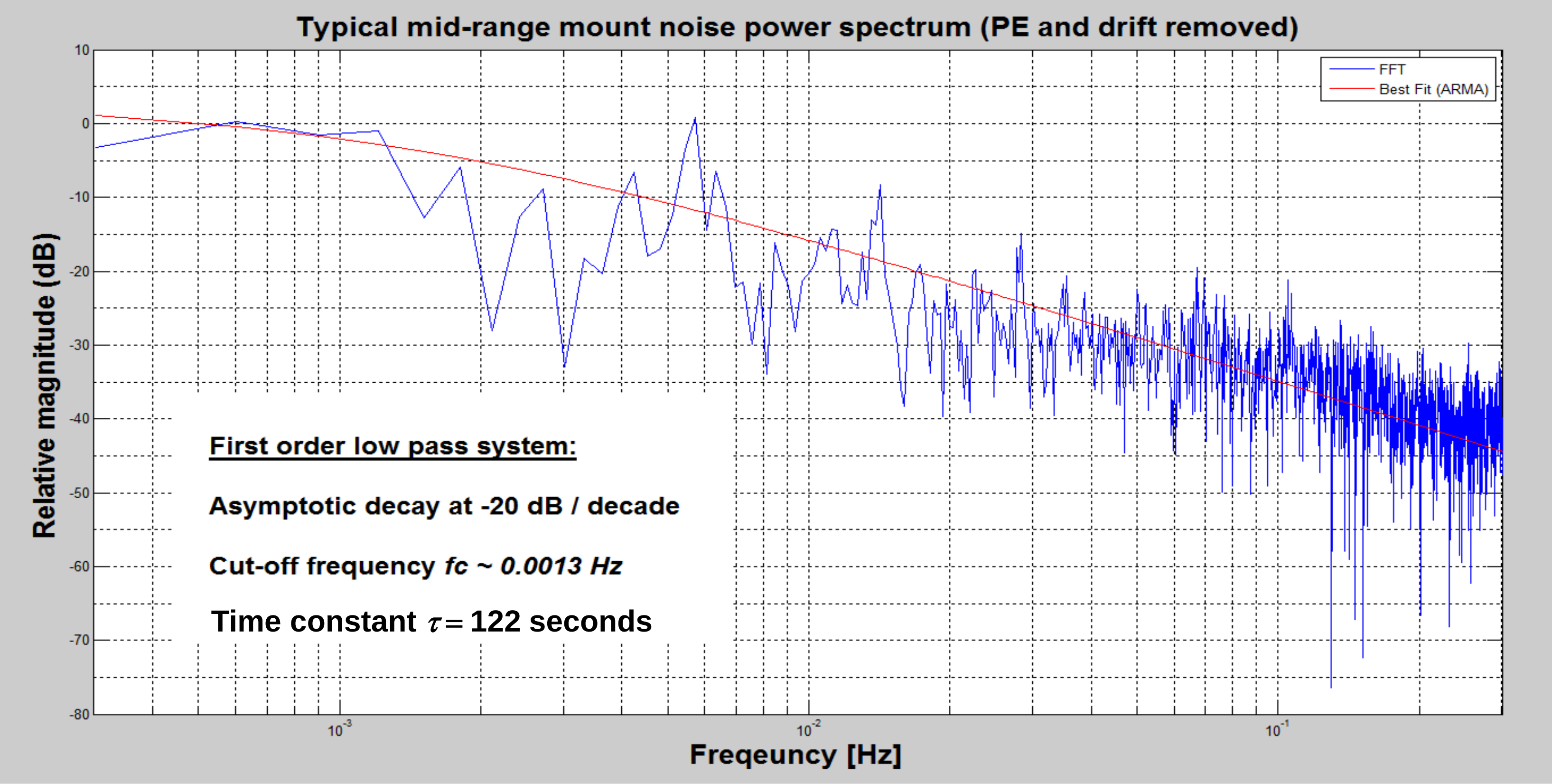
Random setup tracking errors (open loop)

- Mount gear mechanical noise after PEC:
  - Random errors ~0.1” to 1” rms (bandwidth ~ 0.001Hz)
  - Minimized with a good mount (almost gone with direct drive and/or high resolution encoders), auto-guiding
- Wind burst, accidents (bumping mount, cables, mirror flop, …):
  - Temporary or permanent (hysteresis) step like errors
  - Minimized by dropping frames, auto-guiding
- Unforeseen (Mr. Murphy is very creative and works in team)
  - Minimized by dropping frames, auto-guiding
- All of those errors are **fully correlated across the all FOV**!

Mount mechanical noise (after PEC, no drift)

- Low frequency (“pink”) noise (RA in the plot below) (almost gone with high resolution encoders and/or direct drive)



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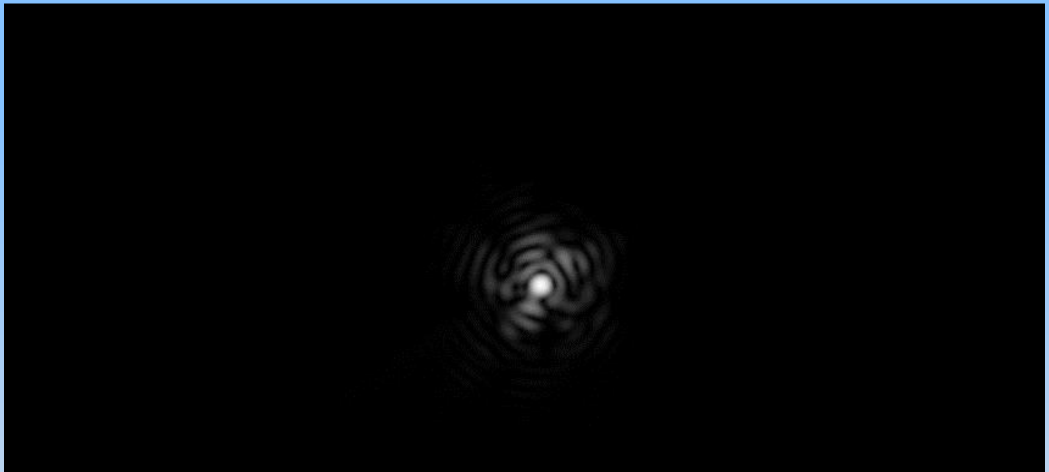
Seeing limited conditions

- Astronomical seeing is the blurring of astronomical objects caused by Earth's atmosphere turbulence
- It impacts the intensity (scintillation) and the shape (phase) of the incoming wave front
- Scintillation is usually not a major problem, at least for exposures above one second.
  Phase is the main concern, mainly wandering stars, since the wavefront tilt/tip contribution >85% of the total seeing phase variance



Star under seeing limited condition (short exposure << 1s)
Credit: John Hayes

The Fried's parameter

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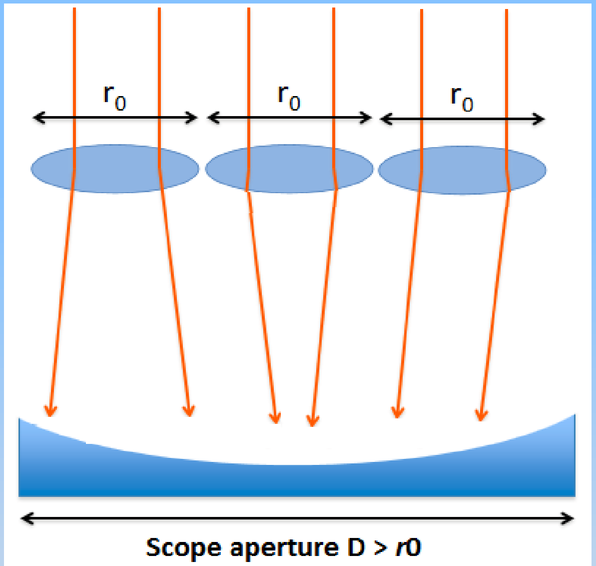
The Fried's parameter r_0 is the **average turbulence cell size**

$$r_0 = \left[\frac{1}{0.423\left(\frac{2\pi}{\lambda}\right)^2 \sec(z)\int_0^\infty C_N^2(h)dh}\right]^{3/5}$$



z zenith angle, λ the wavelength and $C_N^2(h)$ is the atmospheric turbulence strength at the altitude ***h*.**

FWHM ["]	**1**	**1.5**	**2**	**2.5**	**3**
r_0 [mm/inch]	110 / 4.3	74 / 2.9	56 / 2.2	44 / 1.7	37 / 1.5

Diffraction limited images can only be achieved with aperture sizes no more then few inches! Diffraction limited $\rightarrow \frac{D}{r_0} < 1$

Isoplanatic patch

The angle for which the total wavefront error remains almost the same (~λ/6) is known as the isoplanatic angle:

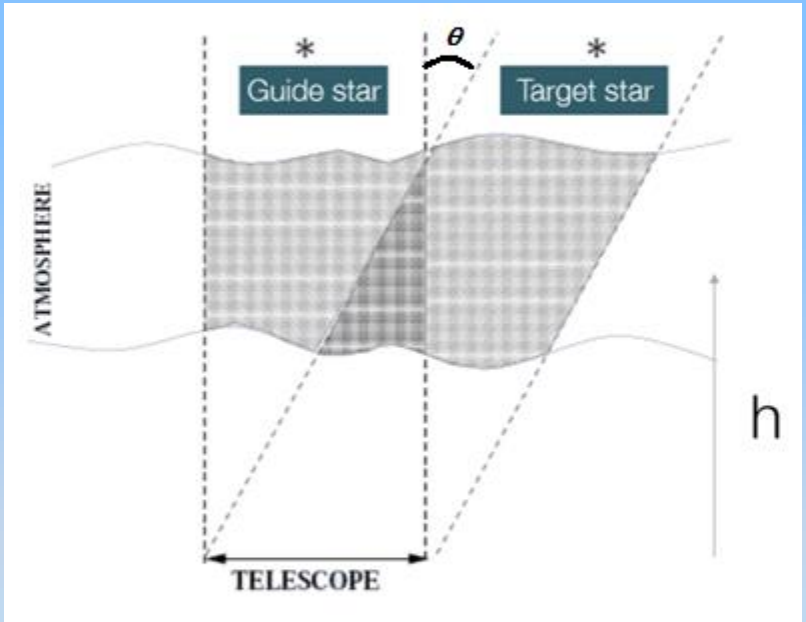
$$\boldsymbol{\theta_0 \cong 0.31\frac{r_0}{\bar{h}}}$$

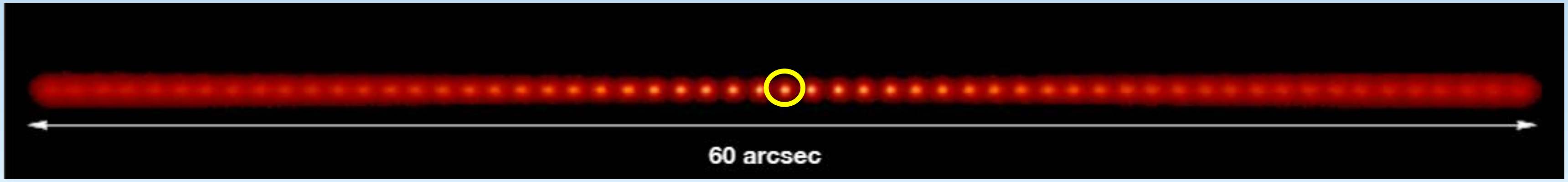
$\bar{h}$ ~ 5km, θ_0 is usually few arc-second across (@550nm):

r_0 = 50mm → ~ 0.6″

r_0 = 200mm → ~ 2.6″

θ_0 increases as $\lambda^{6/5}$





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Isokinetic patch

- The angle for which the wavefront tilt/tip component error remains almost the same is known as the isokinetic angle:

$$\boldsymbol{\theta_m \cong 0.31\frac{D}{\bar{h}}}$$

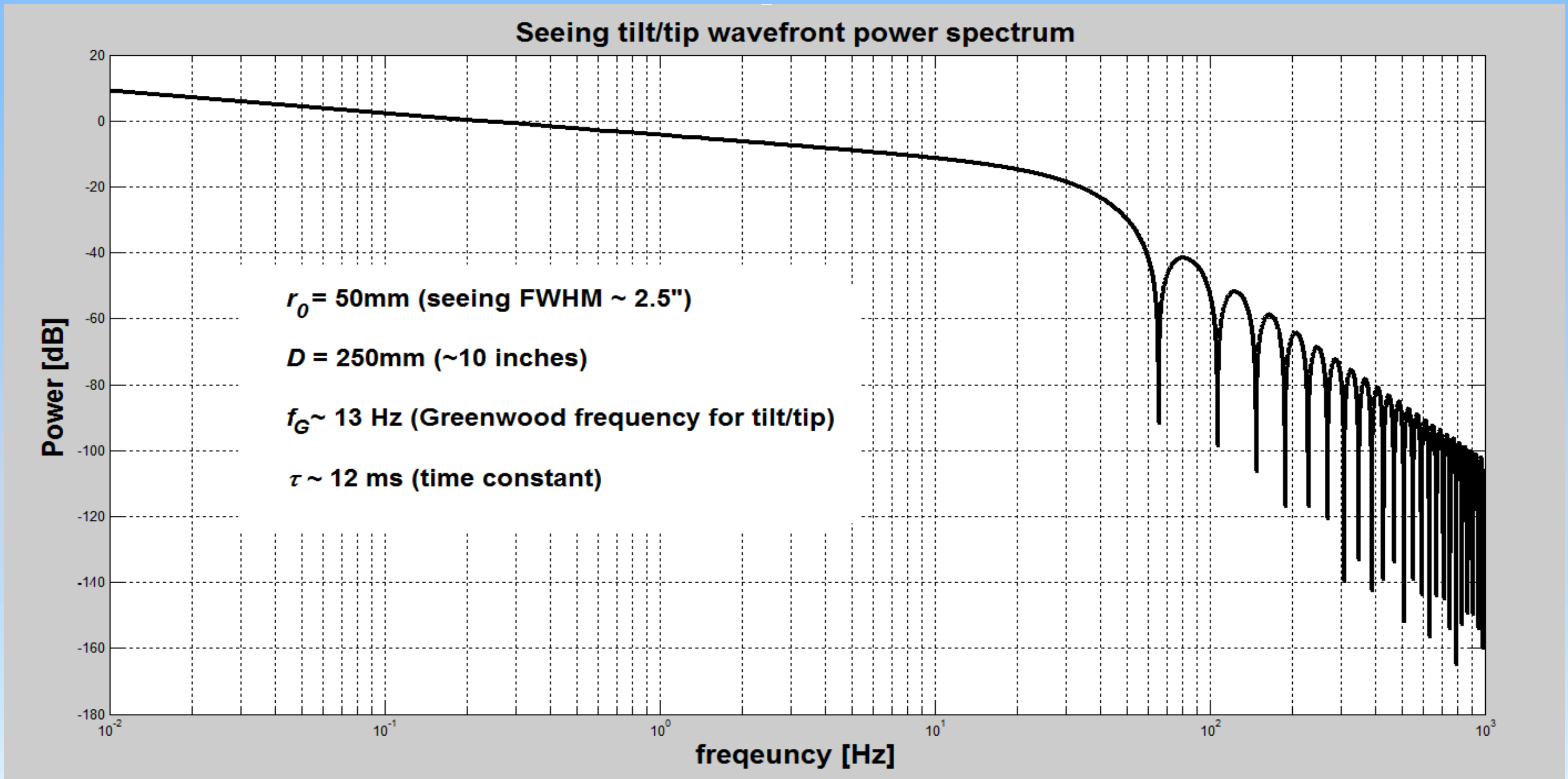
$\boldsymbol{\bar{h}}$ ~ 5km, $\boldsymbol{\theta_m}$ is still few arc-second across:

$\boldsymbol{D}$ = 200mm (~8 inches) → ~ 3″
$\boldsymbol{D}$ = 1m (~40 inches) → ~ 13″

- Conclusions:
  For most setups the seeing **is not correlated across the FOV**! (unless you have a very narrow FOV, arc-second wide)
- → **Guide star behavior is not correlated with the target**

Seeing wave-front tilt/tip (wandering star) power spectrum

- The wave-front tilt/tip seeing component is the dominant effect
- The tilt/tip component is a large ("white") bandwidth noise



The different types of noise

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- A noise is defined by its distribution (Gaussian, Poisson, …), its bandwidth $\boldsymbol{B_n}$ [Hz] and its rms value $\boldsymbol{\sigma_{noise}}$ (noise mean = 0)
- A "white" noise has a very large bandwidth $\boldsymbol{B}$ relative to the system bandwidth $\boldsymbol{B_s}$, hence $\boldsymbol{B_n} \gg \boldsymbol{B_s}$. There is no correlation, nor predictability, between any sample
- A "pink" noise has a narrow bandwidth $\boldsymbol{B_n}$ relative to the system bandwidth $\boldsymbol{B_s}$, hence $\boldsymbol{B_n} < \boldsymbol{B_s}$. There is some level of correlation/predictability between samples
- Seeing, electronic, thermal & "shot" noise are often "white" noise, but they are either weakly or not at all correlated across the FOV
- Mount mechanical noise is usually a "pink" noise fully correlated across the all FOV

Mechanical and seeing noise bandwidths B_n

- The mechanical noise bandwidth is typical ~0.001Hz, or less, while the seeing (tilt/tip) noise bandwidth is ~10Hz, or more, a ratio ~**10,000x**
- Both noises have different consequences for auto-guiding
- For guider exposures (sampling periods) $\sim \mathbf{0.1}\boldsymbol{s} \leq \boldsymbol{\Delta t} \leq \mathbf{30}\boldsymbol{s}$:

-> *Sampled seeing noise remains an unpredictable "white" noise under all seeing conditions (good or poor):*

→ Can not be corrected

-> *Sampled mechanical noise remains a partially predictable "pink" noise, samples are similar from one to the next:*

→ Can be corrected

Total open loop noise (PEC, accidents & drift removed)

- The mount mechanical noise and seeing noise are uncorrelated to each other, their variances $\boldsymbol{\sigma_m^2}$ and $\boldsymbol{\sigma_s^2}$ add in quadrature.
- Therefore the total tracking noise variance $\boldsymbol{\sigma_{total}^2}$ (open loop) is:

$$\boxed{\boldsymbol{\sigma_{total}^2 = \sigma_m^2 + \sigma_s^2}}$$

- The total tracking noise rms $\boldsymbol{\sigma_t}$ is then:

$$\boxed{\boldsymbol{\sigma_{total} = \sqrt{\sigma_m^2 + \sigma_s^2}}}$$

Auto-guiding error (close loop) on a target

- The classical auto-guiding strategy calls for a mount (or AO-tilt/tip) correction $\boldsymbol{c}[\boldsymbol{n}]$ proportional to the guiding error $\boldsymbol{e}[\boldsymbol{n}]$. At the $\boldsymbol{n}^{th}$ guider frame the close loop correction is:

$$\boxed{\boldsymbol{c}[\boldsymbol{n}] = -\boldsymbol{K}\boldsymbol{e}[\boldsymbol{n}]}$$

- $\boldsymbol{K}$ is known as the "aggressiveness", usually $0 \leq \boldsymbol{K} \leq \mathbf{1}$
- The guiding error (close loop) impacts the target image quality
- The guiding error is function of mount/setup error & seeing
- There are two basic parameters ("knobs") to control it:

1. Guider exposure time $\Delta \boldsymbol{t}$ = correction period, usually
2. Aggressiveness $\boldsymbol{K}$ (one for RA and one for DEC)

Understanding the auto-guiding

(proportional control)

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- We can use the Z-transform to derive the transfer function $\boldsymbol{H(Z)}$ of a digital control system, which is similar to the **MTF** in an optical system. It describes how a digital control system responds at disturbances at different temporal frequencies

$$\boxed{\boldsymbol{H(Z) = \frac{E(Z)}{D(Z)} = \frac{1 - Z^{-1}}{1 - (1 - K)Z^{-1}}}}$$

- $\boldsymbol{H(Z)}$ relates any disturbance/perturbation $\boldsymbol{D(Z)}$ applied to the mount/setup to the close loop error $\boldsymbol{E(Z)}$, after correction
- $\boldsymbol{H(Z)}$ is the same for any disturbance, drifts, steps or noises, therefore the Z-transform is an universal tool, like the **MTF** is.

Auto-guiding system stability
(step response diverged)

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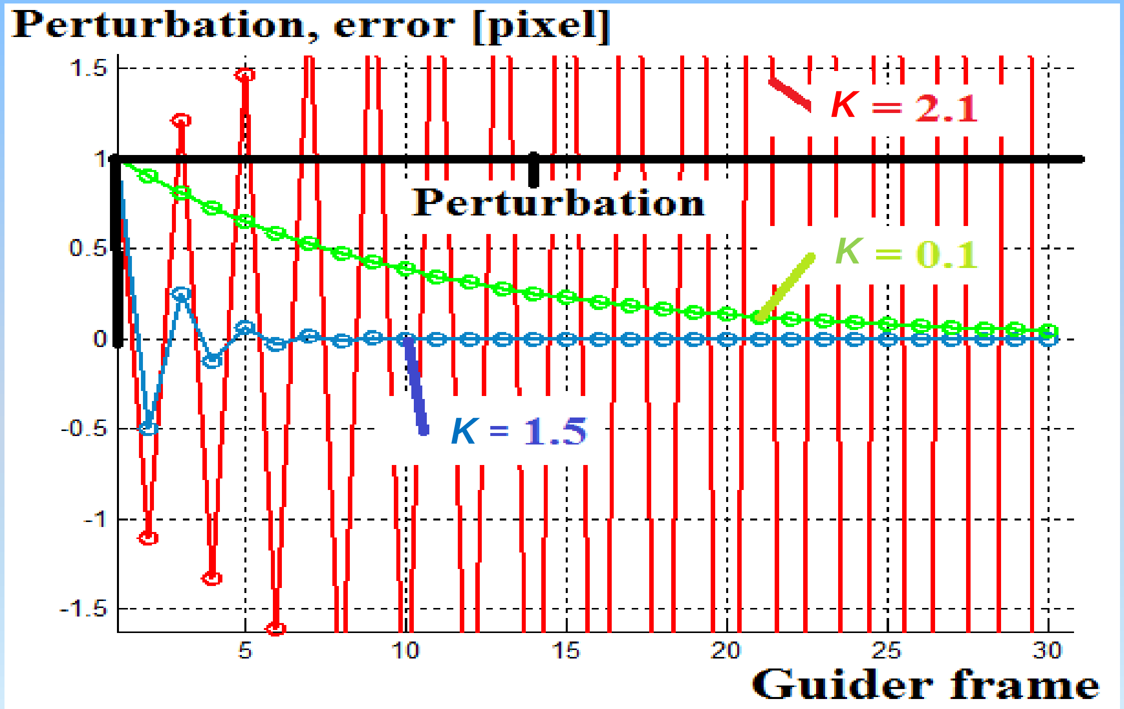
- $H(Z)$ stable without overshoot for $\mathbf{0 \leq K \leq 1}$
- $H(Z)$ stable **with overshoot for** $\mathbf{1 < K \leq 2}$
- $H(Z)$ **unstable for** $\mathbf{K > 2}$



Auto-guiding analysis
3 basic situations

- To understand how a basic auto-guiding algorithms acts on error let's analysis $H(Z)$ reponse for 3 classical perturbations.

- <u>Step response:</u>
  A one time perturbation, a "bump" (no noise, deterministic).

- <u>Drift response:</u>
  A constant drift perturbation (no noise, deterministic).

- <u>Noise response</u>:
  A random perturbation, "white", or "pink" noise (average = 0)

<u>P.S:</u> Under the linearly assumption the superposition theorem holds.
The total response is the sum of the individual responses.

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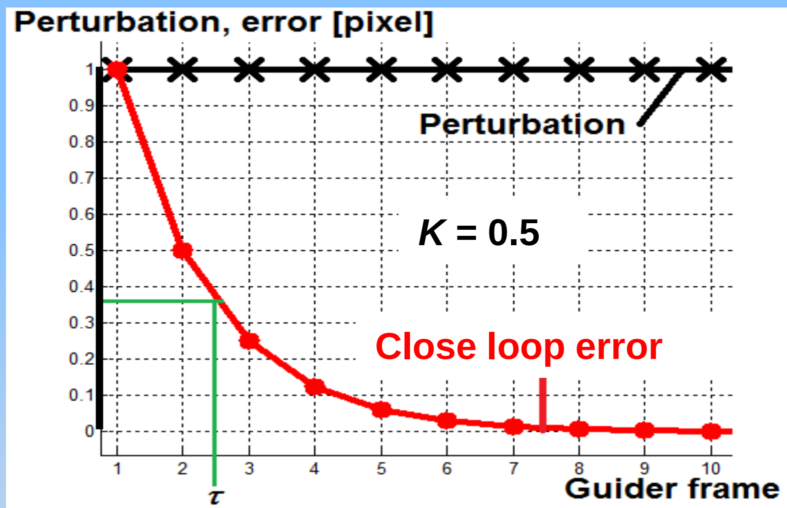
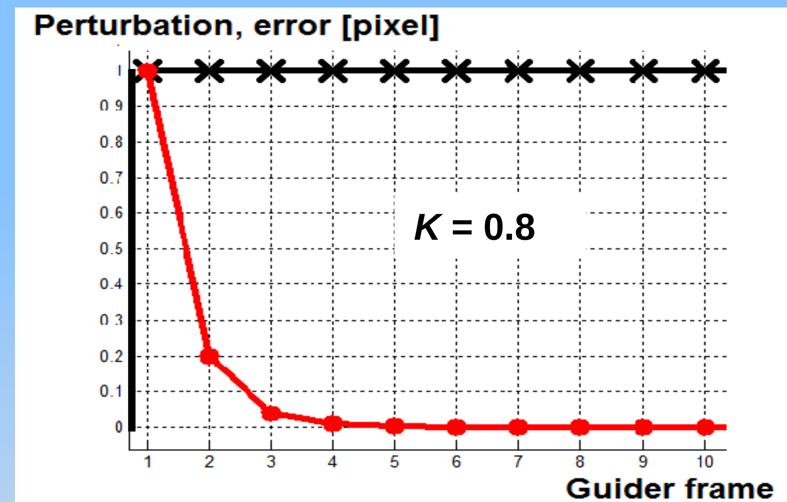
Auto-guiding
The step response

- The plots below show the typical step response (no noise):





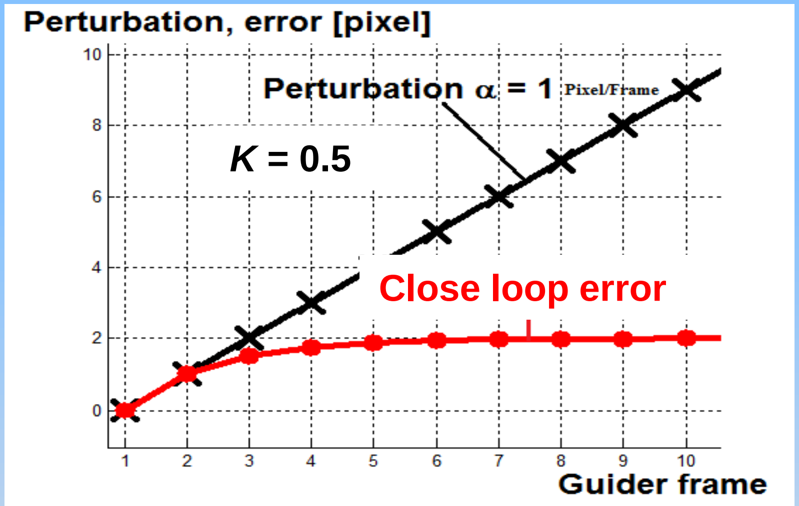
- $e[n]$ decays exponentially from guider frame to frame (n).
- The error decayed by ~63% after one time constant τ :

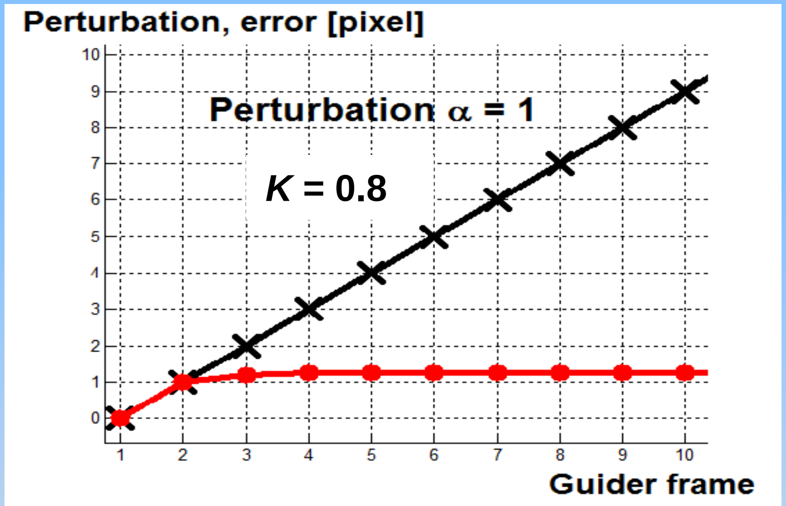
$$\boxed{\tau = \frac{-\Delta t}{ln(1-K)}}$$

Δt = auto-guiding period, ex. P = 1 px, Δt = 2s, K = 0.5, $\tau \cong$ 2.9s

Auto-guiding
The drift response

- The plots below show the typical drift response (no noise):





- $\boldsymbol{e}[\boldsymbol{n}]$ increases with $\boldsymbol{n}$, then settles. Same $\boldsymbol{\tau}$ than for a step
- The final close loop error $\boldsymbol{e}[\boldsymbol{n} \to \infty]$ is (a constant bias):

$$\boxed{\mathbf{e}[\boldsymbol{n} \to \infty] = \frac{\propto}{\boldsymbol{K}}}$$

$\propto$= drift during $\boldsymbol{\Delta t}$, ex.$\propto = \mathbf{1}\ \frac{\mathbf{pixel}}{\boldsymbol{\Delta t}}$, $\boldsymbol{K} = 0.5$, $\boldsymbol{e}[\infty] = 2$ pixels

Auto-guiding
The "white" noise response

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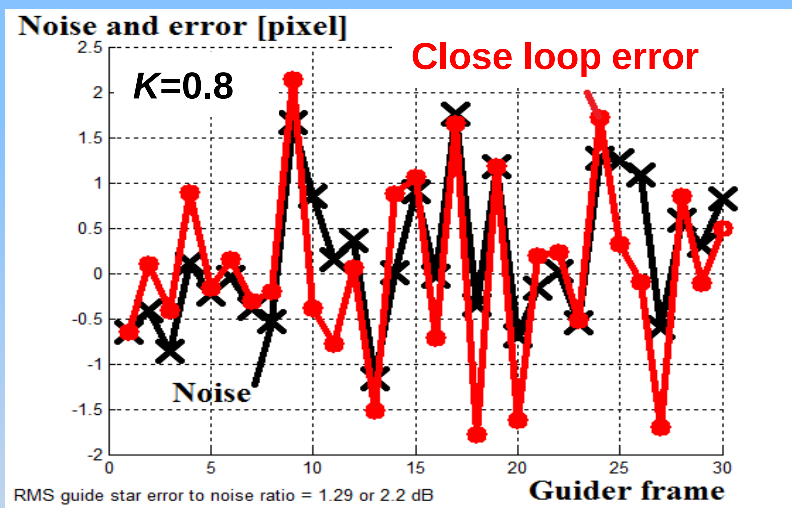
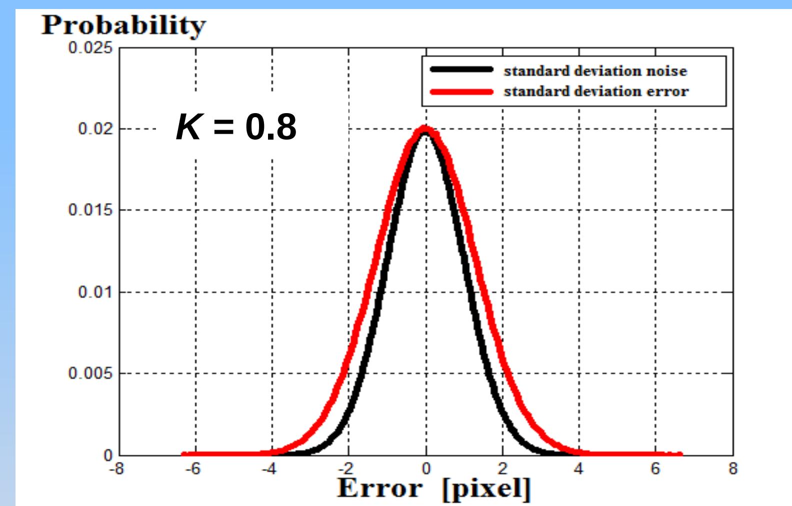
- The plots below show the response to a "white" (broadband) noise of variance $\boldsymbol{\sigma_n^2}$ ($\boldsymbol{\sigma_n}$ = rms value):





The error is a noise too with $\sqrt{2}\boldsymbol{\sigma_n} \geq \boldsymbol{\sigma_e} \geq \boldsymbol{\sigma_n}$, its rms value is:

$$\boxed{\boldsymbol{\sigma_e = \sqrt{\frac{2}{2-K}}\,\sigma_{noise}}}$$

ex. $\sigma_{noise} = 1$ pixel (rms), k = 0.8, $\sigma_e \cong$ 1.29 pixels

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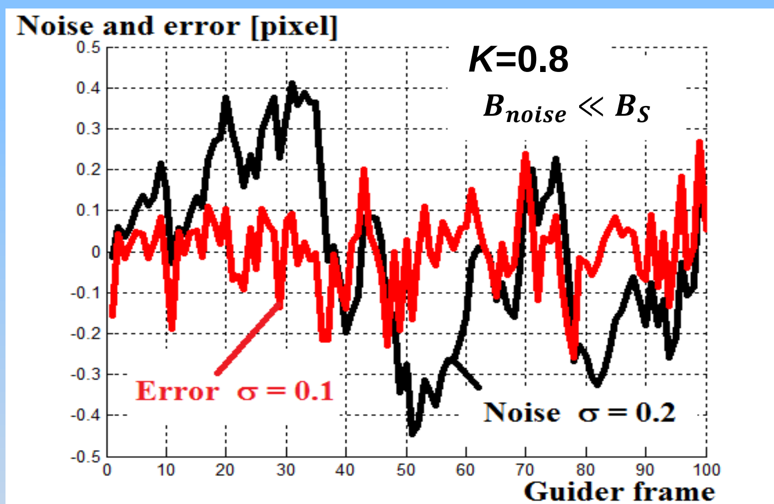
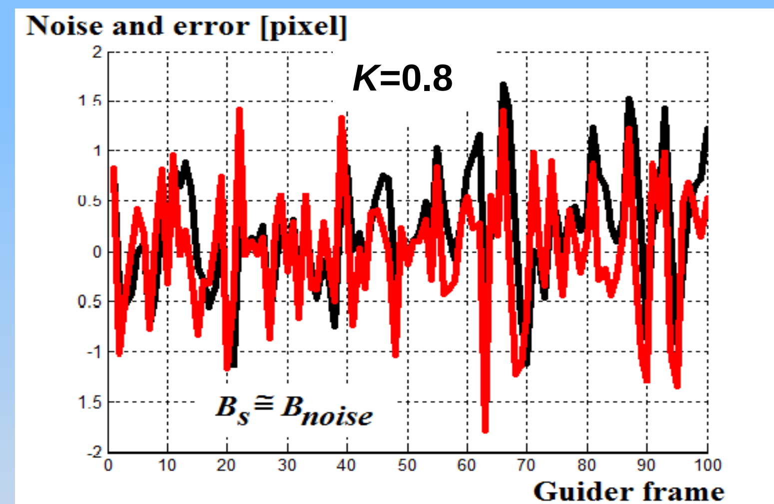
Auto-guiding
The "pink" noise response

- The plots below show the response to a typical "pink" noise of variance σ_n^2 (σ_n = rms value):





- The stronger the noise correlation the smaller the close loop error for the same K.
- The mathematics are more complex than for a "white" noise but trackable.

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Optimal auto-guiding

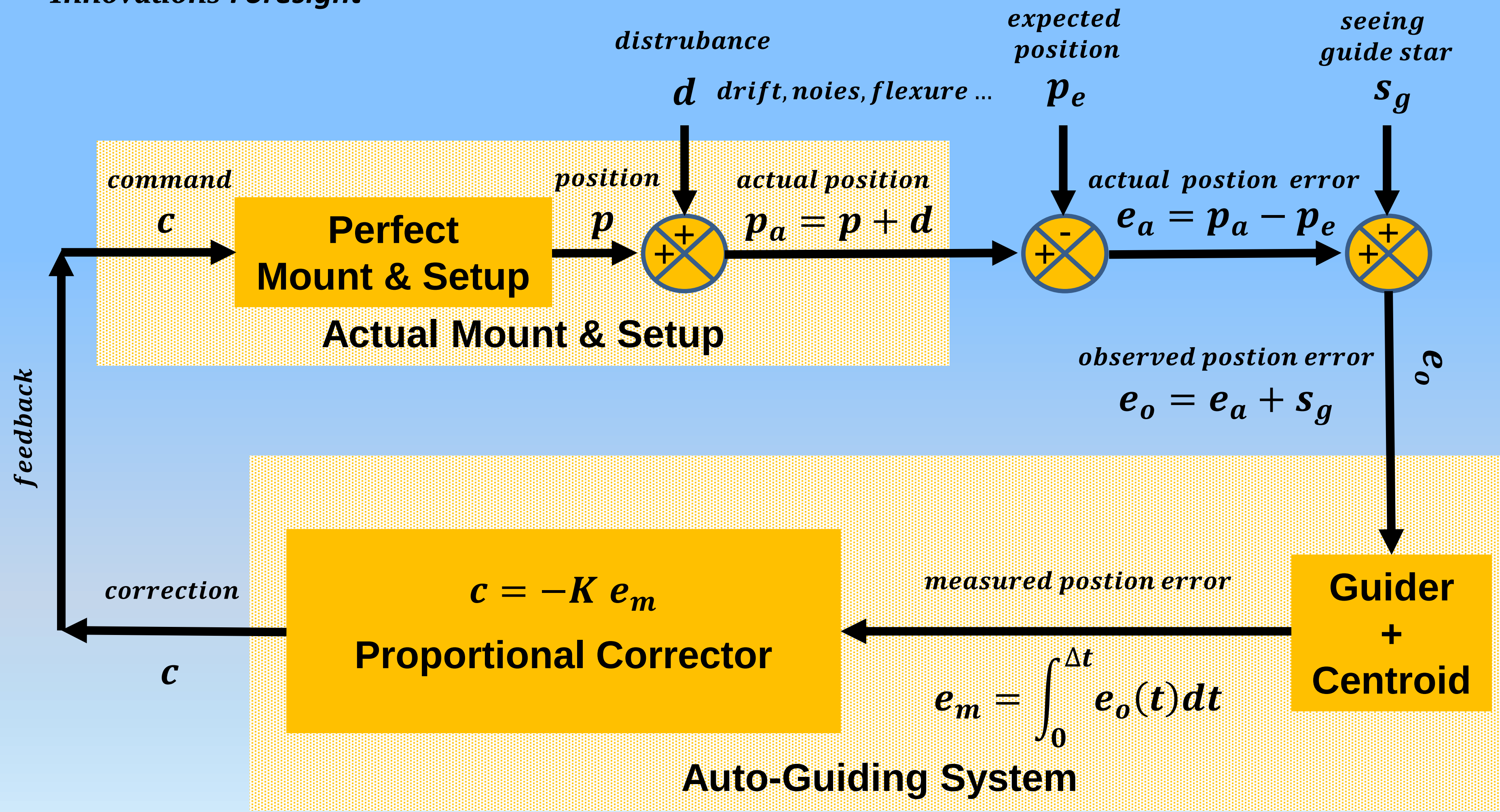
- Stepwise perturbations are eventually fully corrected with a time constant $\boldsymbol{\tau} = \frac{-\Delta t}{ln(1-K)}$, usually few guider frames.
- Drift perturbations eventually settle to a quasi constant close loop error within $\boldsymbol{\tau} = \frac{-\Delta t}{ln(1-K)}$, usually few guider frames.

  Drift rate changes slowly & is typically just a bias (close loop).
- <u>Noises are the main concern</u>

  Optimal auto-guiding aims at **minimizing the total close loop noise rms value $\boldsymbol{\sigma_t}$ on a target**, in other words:

> Given a mount performance $(\boldsymbol{\sigma_m}, \boldsymbol{f_c})$ & local seeing $(\boldsymbol{\sigma_s}(\boldsymbol{D}, \boldsymbol{r_0}, \boldsymbol{\lambda}, \boldsymbol{f_G}))$ what should be the best $\boldsymbol{\Delta t}$ and $\boldsymbol{K}$ values for minimizing $\boldsymbol{\sigma_t}$?

Auto-guiding loop: The big picture



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Target error on imager
Final FWHM

seeing target s_t

actual postion error e_a

mount close loop error

target terror $e_t = e_a + s_t$

Imager

$FWHM \cong 2.36\sqrt{\sigma_{e_a}^2 + \sigma_{s_t}^2}$

seeing limited

- Assumptions/Validity:
  - Imager exposure time >> mount time constant >> 1 minute typically
  - Guider exposure time ~ $0.1s \leq \Delta t \leq 30s$
  - Seeing limited condition $\rightarrow D > r_0$
    - *Under average seeing 2.5″*$\rightarrow D > 50mm$
  - Target outside the guide star isokinetic patch $\theta_m = 0.31\frac{D}{h}$
  - *Under average seeing 2.5″*$\rightarrow \theta_m \cong 13D$ ["], *D in meter*
- Mount close loop and target seeing errors add in quadrature

Effect of guider exposure Δt for $0.1s \leq \Delta t \leq 30s$

- The guider sensor integrates (averages) the noise during exposure. Acting as a low pass filter with cut-off frequency f_i:

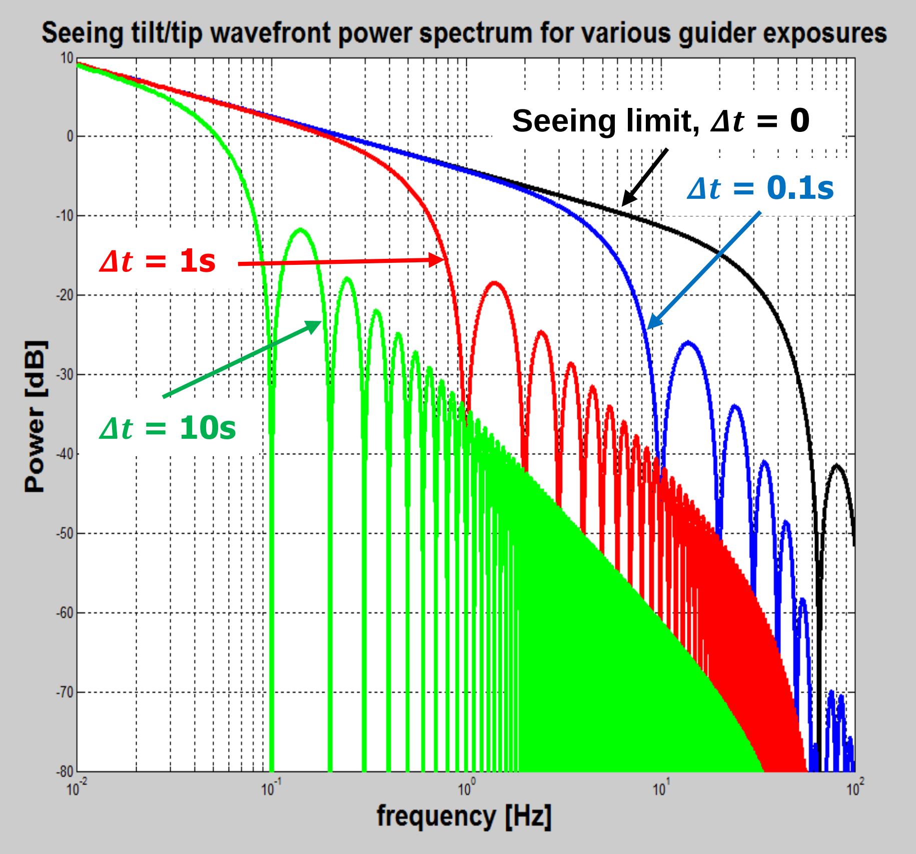
$$\boxed{f_i = \frac{1}{\pi \Delta t} \cong 0.32 \cdot \frac{1}{\Delta t} \quad [Hz]}$$

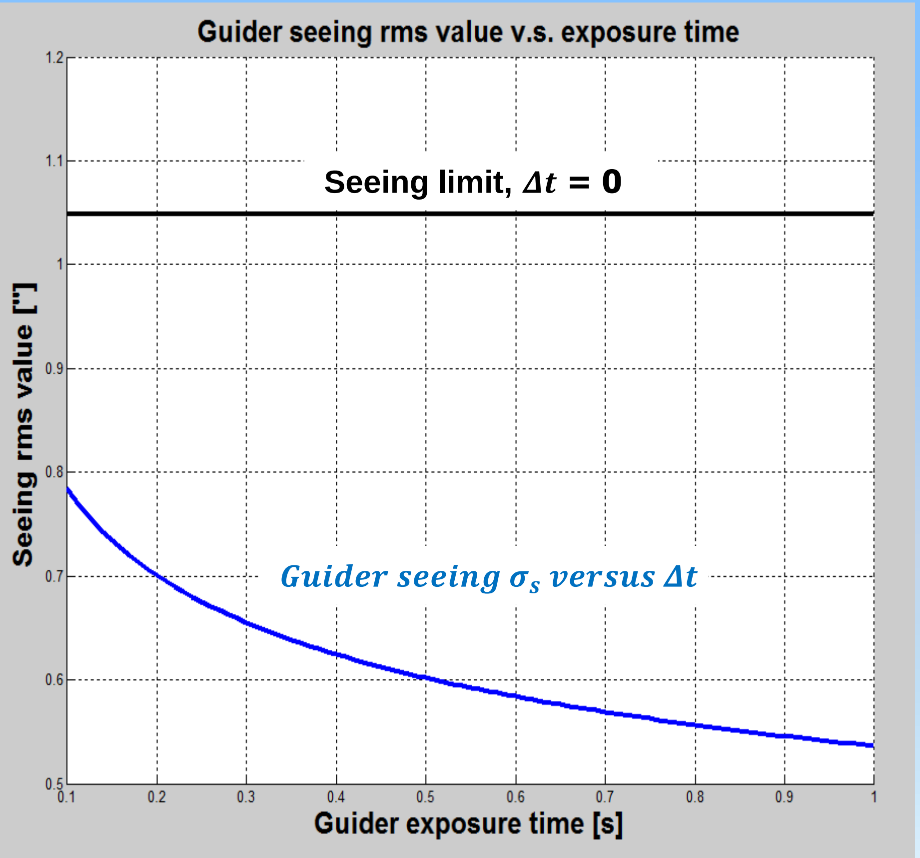
$$\boxed{0.1s \leq \Delta t \leq 30s \rightarrow 3Hz \geq f_i \geq 0.01Hz}$$

- Mechanical noise bandwidth is typical around 0.001Hz, hence essentially left untouched (unfiltered) by the guider, $f_c \ll f_i$
- Seeing (tilt/tip) noise bandwidth is typically around 10Hz, or more, hence low pass filtered by the guider, $f_G \gg f_i$
- Those two very different bandwidths provide a way to filter the seeing, which cannot be corrected, while correcting, at least partially, the mechanical noise leading to optimal guiding.

Effect of guider exposure Δt on seeing power spectrum

- Longer guider exposures $\boldsymbol{\Delta t}$ lead to lower seeing rms $\boldsymbol{\sigma_s}$ contribution values on auto-guiding $(D = 250mm, r_0 = 50mm, \lambda = 550nm)$





Aggressiveness K and close loop rms error mid-range mount (4" peak-peak, after PEC)

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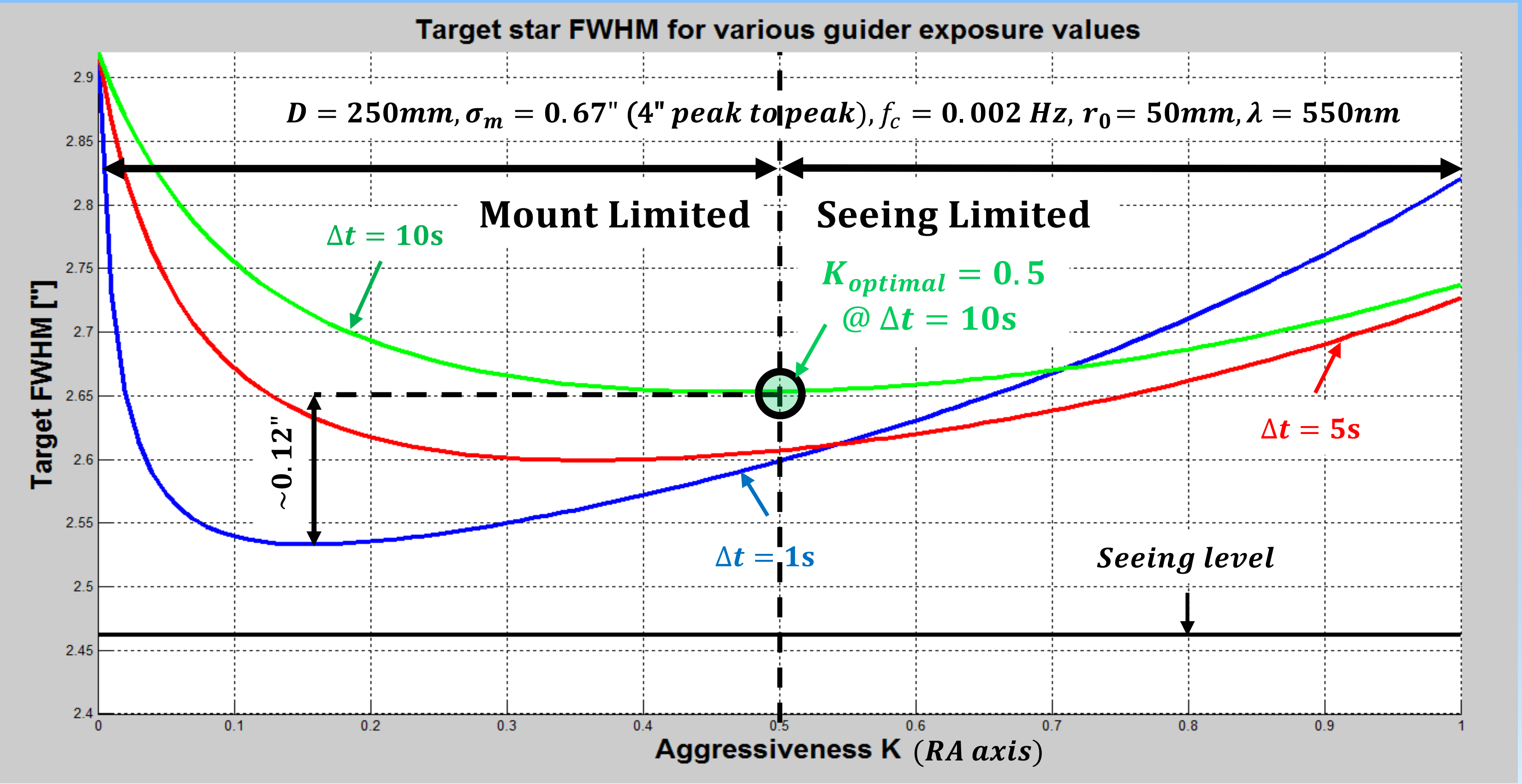
- For a given mount, seeing & Δt, the total close loop noise error rms value σ_t exbibits a minimum value for some $K_{optimal}$



Aggressiveness *K* and close loop rms error high-end mount (1" peak-peak, after PEC)

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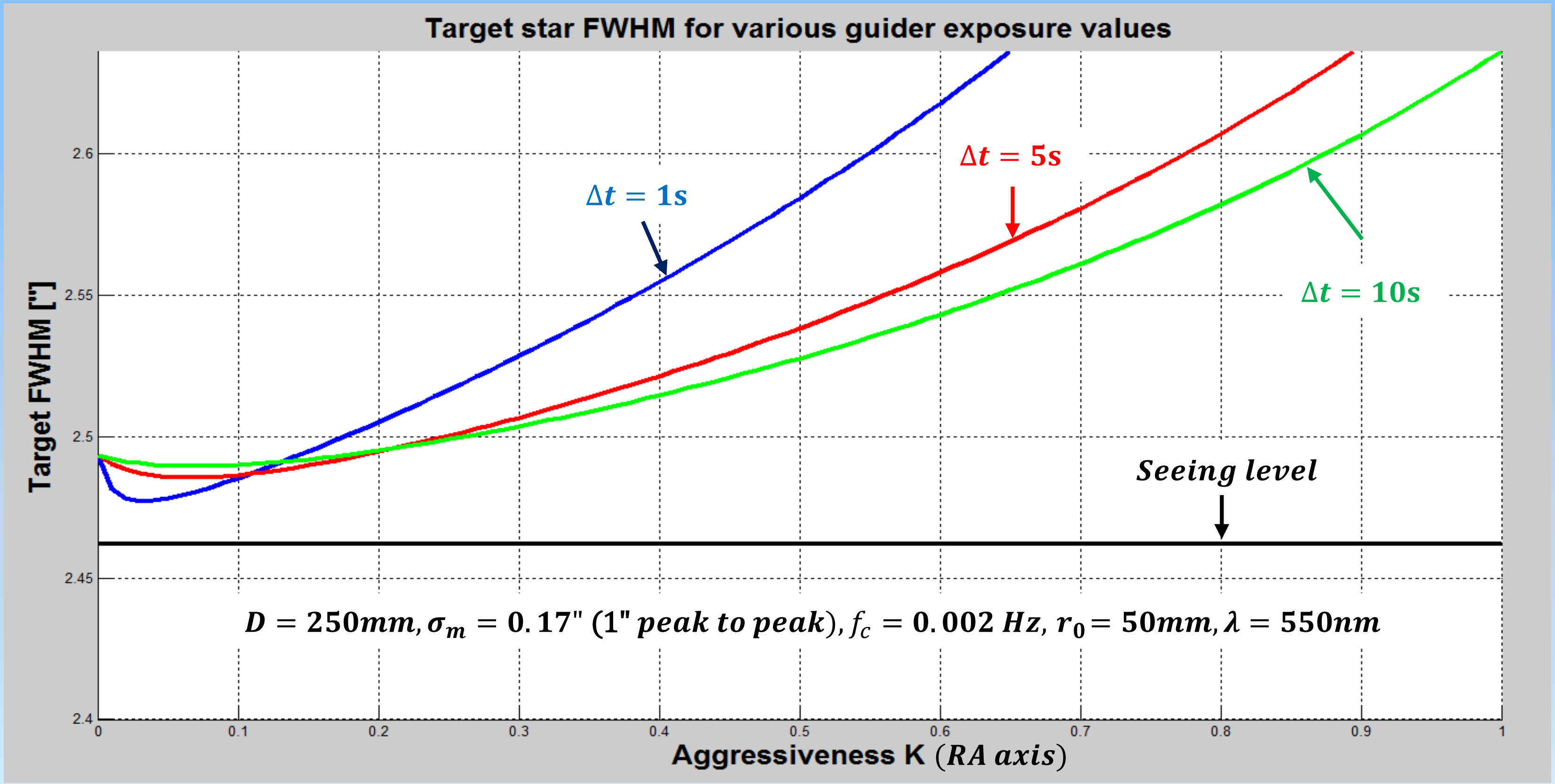
- A lower mount error (σ_m) leads to smaller close loop errors under same seeing. Most guide exposures give the same result.



Target FWHM versus guider exposure time Δt and aggressiveness K

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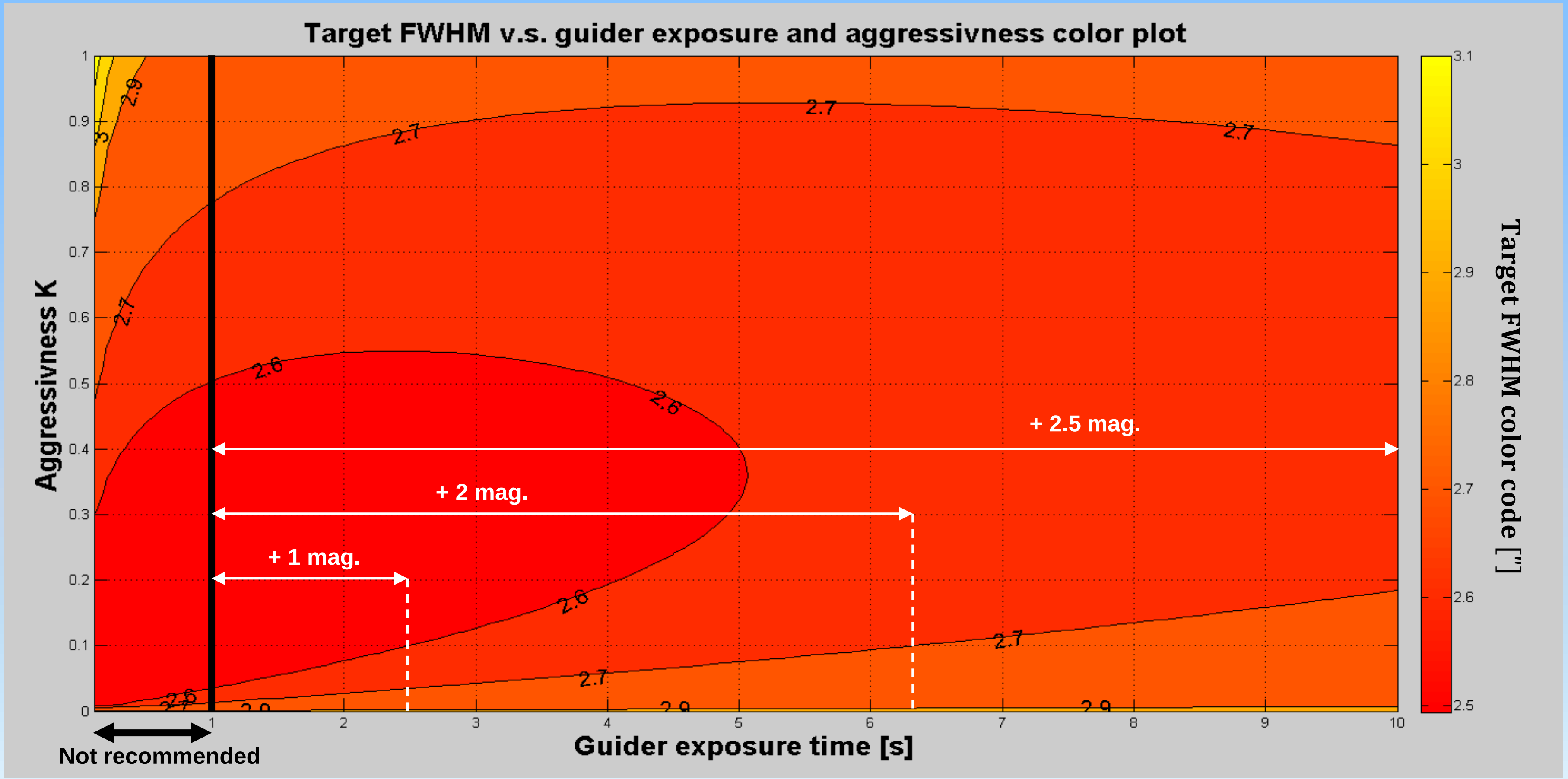
- Seeing = 2.5″, mid-range mount = 4″ peak-peak (after PEC)
- $\Delta t < 1s$ is not recommended (prone to scintillation/aberration)



Open loop seeing error scatter plot

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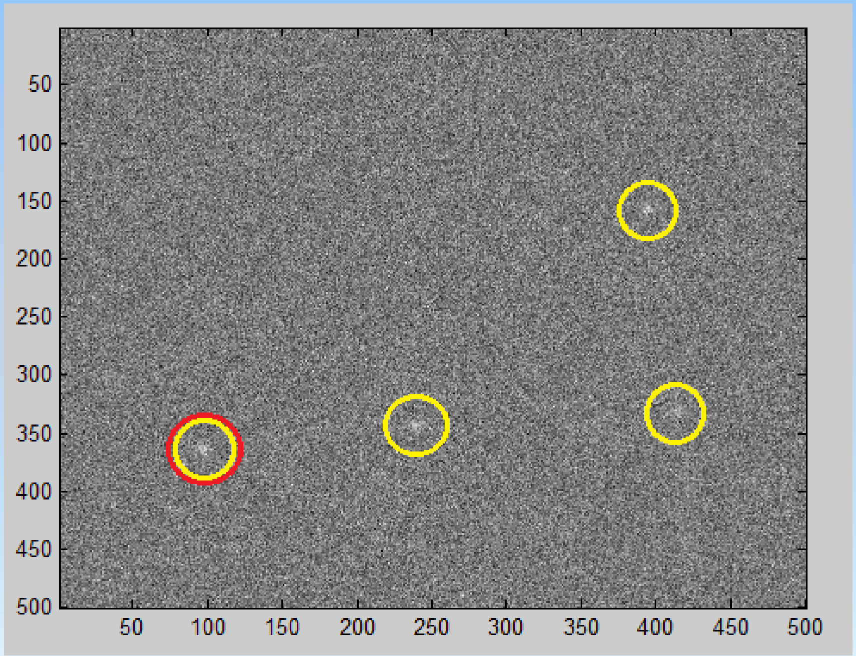
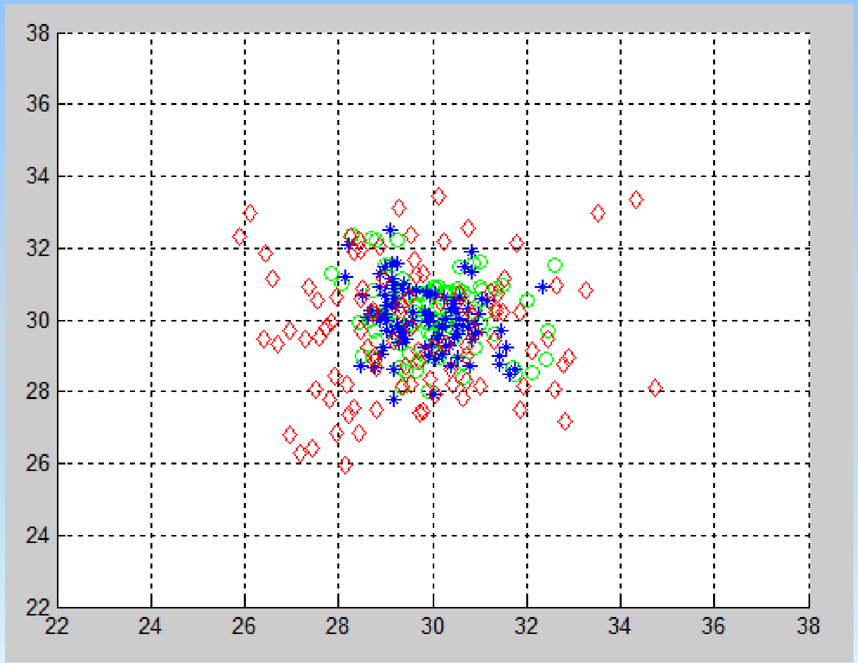
Perfect mount open loop error scatter plot (100 samples).
SNR=6 dB (2x), 4 stars (same mag.), seeing 2 pixel rms.
Red diamond: One star centroid.
Green dot: Full frame guiding ADIC (uses the all frame).
Blue dot: Multi-star centroids (uses 4 star centroids).

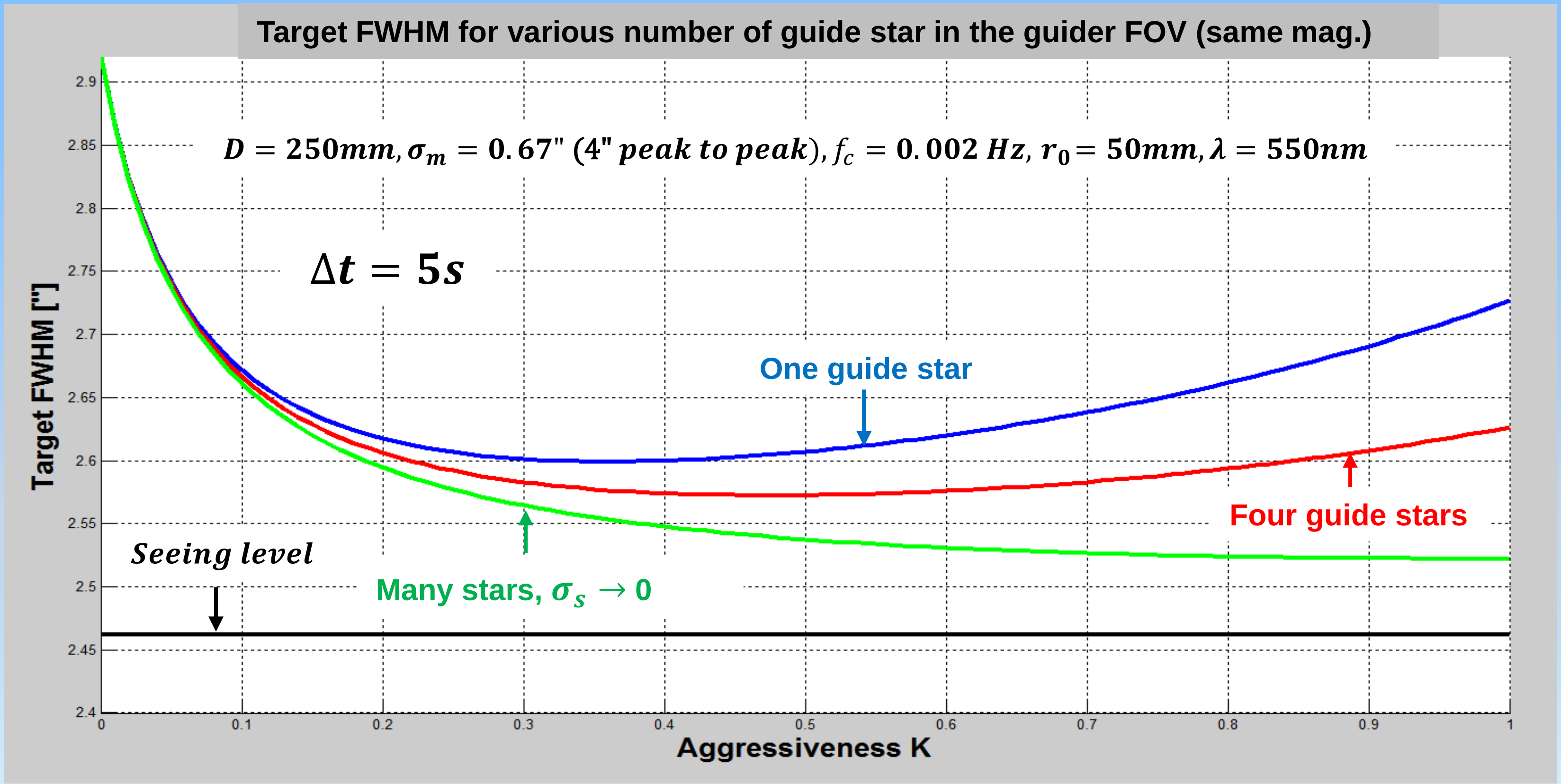




Close loop error versus information in guider FOV

mid-range mount (4" peak-peak, after PEC)

- More information (like many stars) reduces guider seeing rms error σ_s improving target FWHM



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Thank you!

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Clear skies!