Astronomical Seeing

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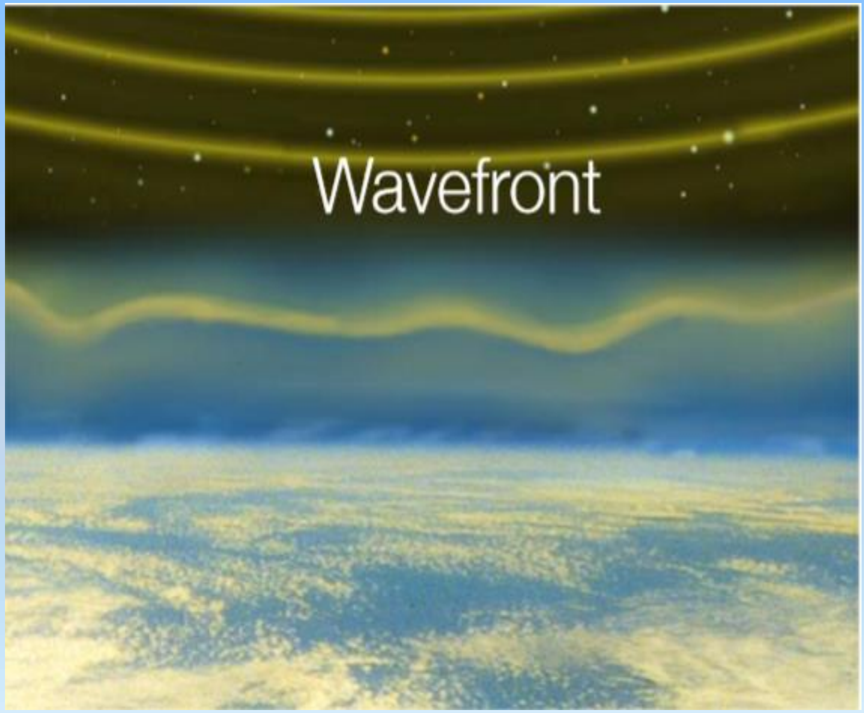
Innovations Foresight, LLC

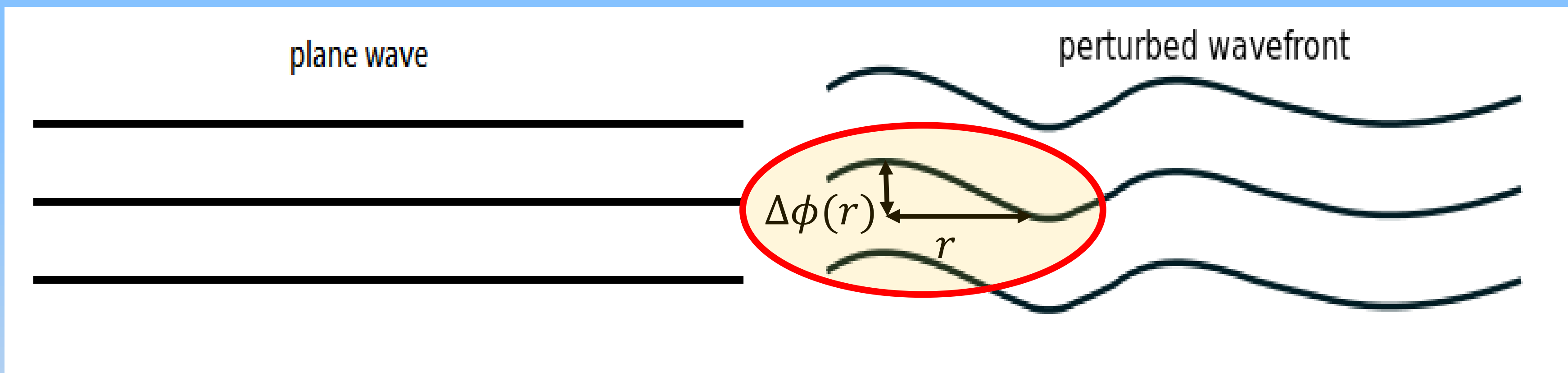
Seeing

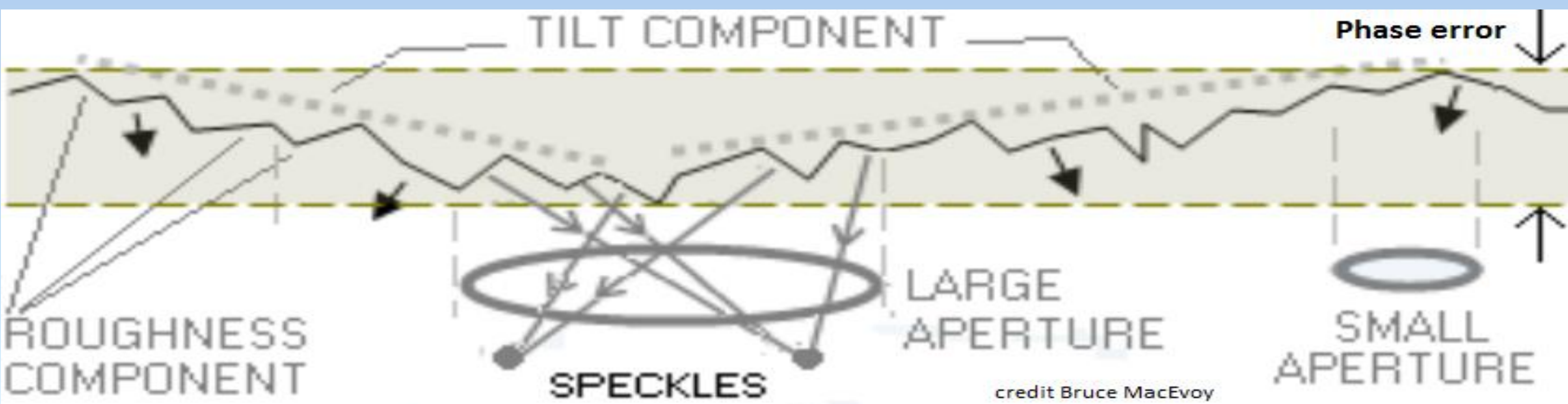
- Astronomical seeing is the blurring of astronomical objects caused by Earth's atmosphere turbulence and related optical refractive index variations (air density fluctuations).
- It impacts the intensity (scintillation) and the shape (phase) of the incoming wave front.
- This presentation will ignore scintillation.



Wavefront and phase distortion

An incoming plane wave is perturbed by the Earth atmosphere turbulent structure leading to phase errors.





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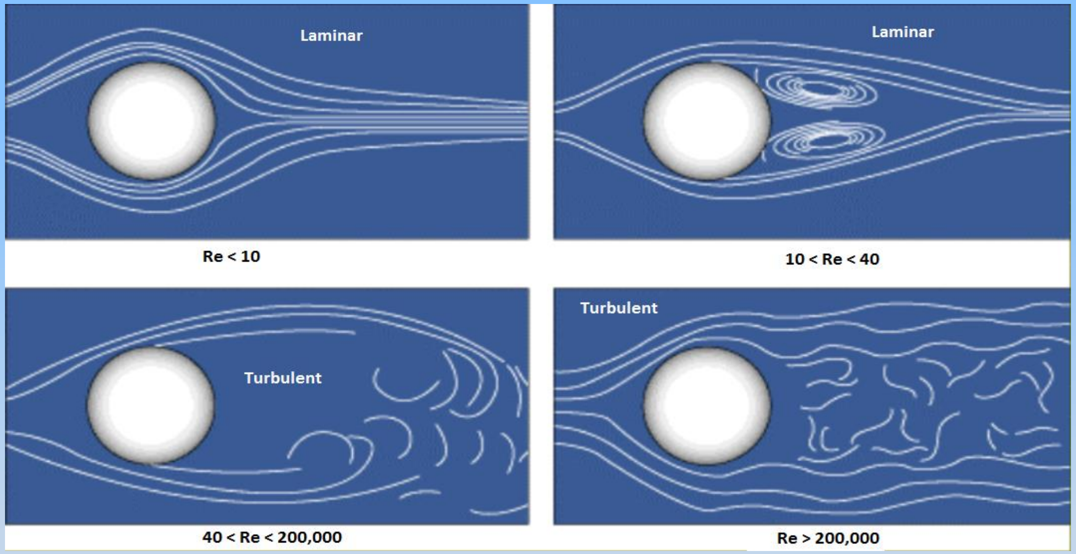
Fluid dynamics

The Reynolds's (O. Reynolds 1883) number Re predicts flow patterns.

$$\mathrm{Re} = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{\rho \mathbf{v} L}{\mu}$$



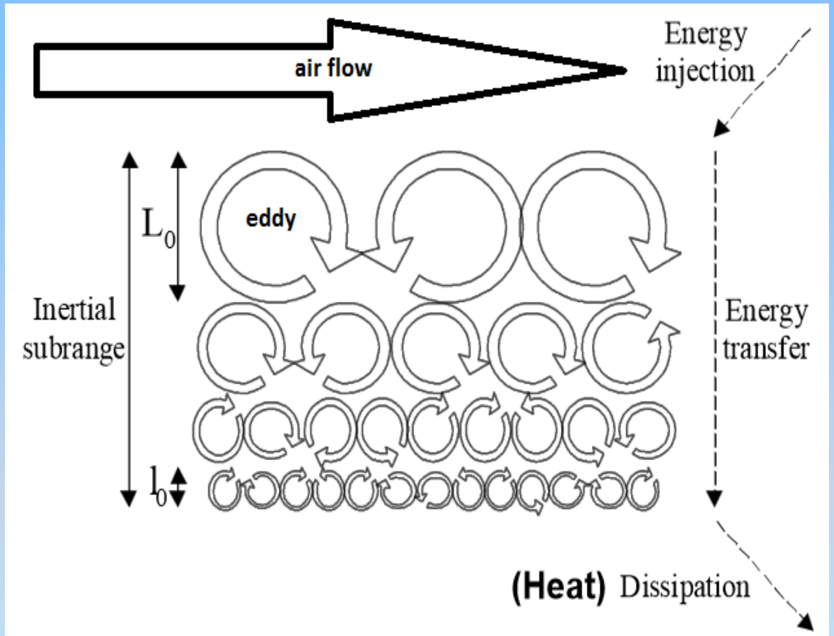
- ρ Density
- $\mathbf{v}$ Velocity
- L Characteristic length
- μ Viscosity

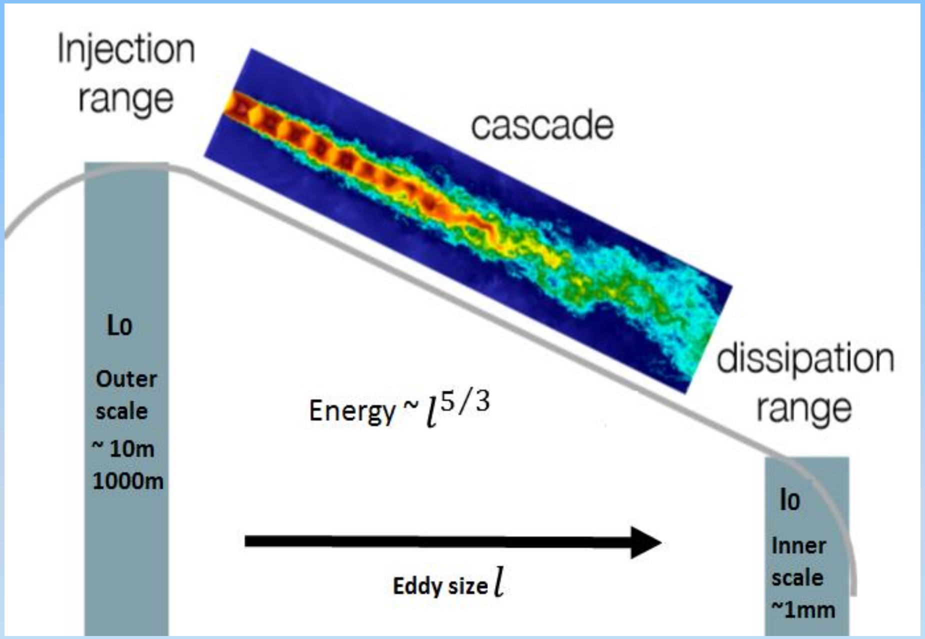
For air flow with velocity of few km/h and for length scales of 0.1km to 1km Re > 10^6.

Therefore the atmosphere is almost always turbulent.

Source of optical turbulence

Energy is transferred by a turbulence cascade form the outer scale L_0 to the inner scale l_0 until heat dissipation (friction).





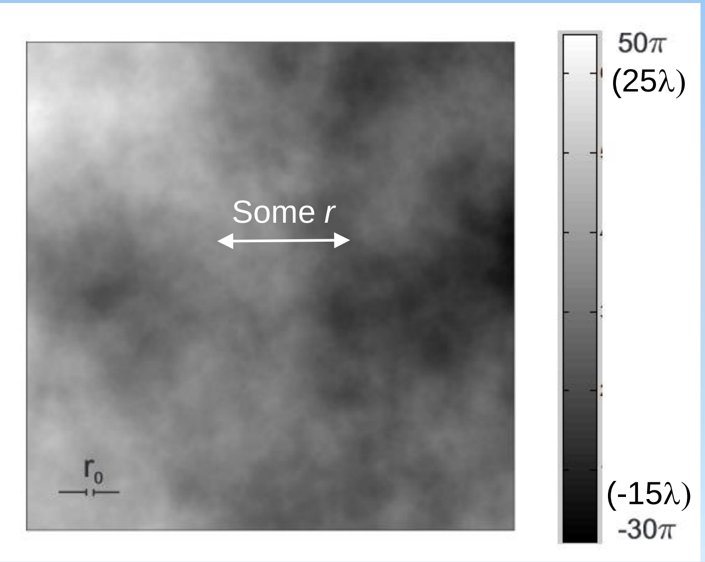
Eddies exhibit different air density -> different refraction index

Phase structure function

The Kolmogorov's turbulence model (A. Kolmogorov 1941) assumes an infinite outer scale turbulence (OST) value L_0.

The related wave front phase error variance, known as the phase structure function, is given by:

$$\boldsymbol{D_{\phi}(r) = \langle \Delta\phi(r)^2 \rangle = 6.88\left(\frac{r}{r_0}\right)^{5/3}}$$

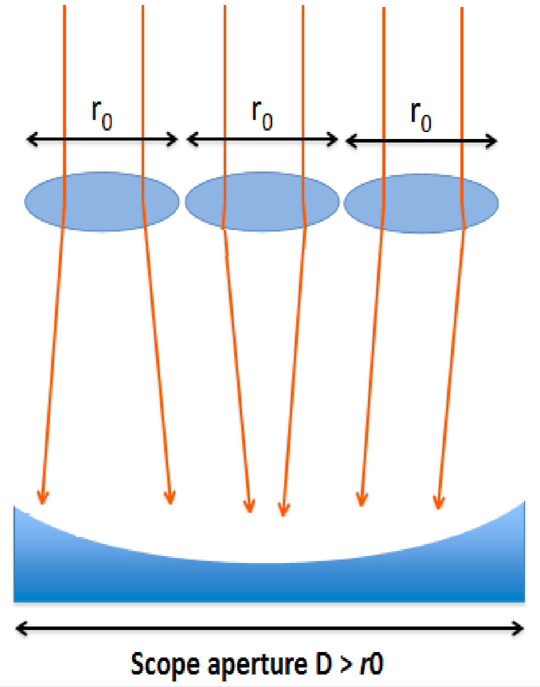


r_0 is known as the Fried's parameter, or coherence length.
(David Fried 1960)

The Fried's parameter

The Fried's parameter r_0 is the **diameter** for which the rms wave front phase error is about 1 radian, or ~λ/6.
It is the "average" turbulence cell size.

$$r_0 = \left[0.423\left(\frac{2\pi}{\lambda}\right)^2 \sec(z)\int_0^{\infty} C_N^2(h)dh\right]^{-3/5}$$

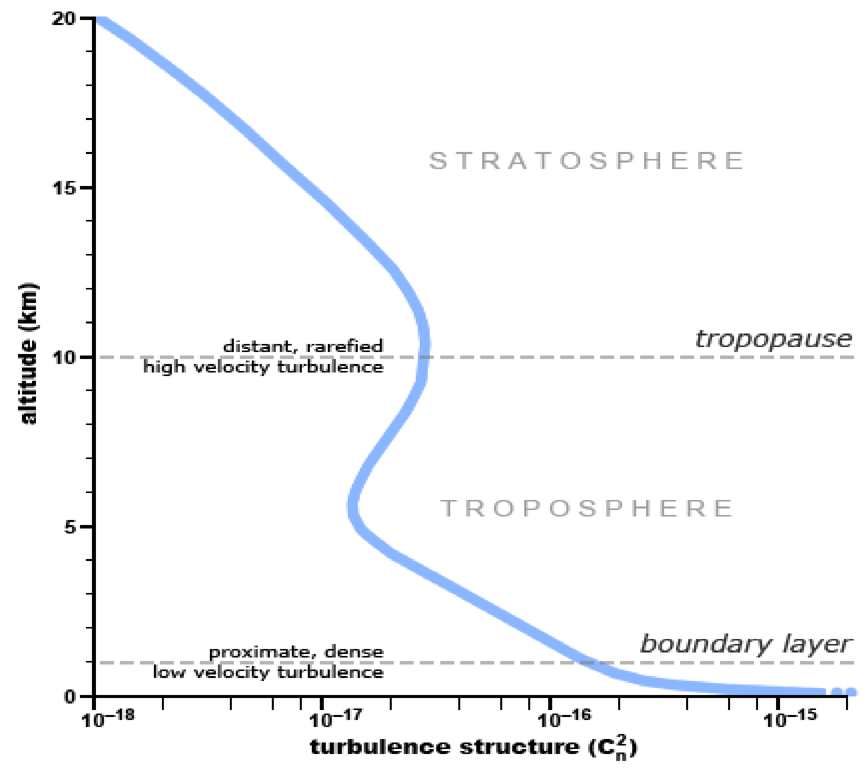


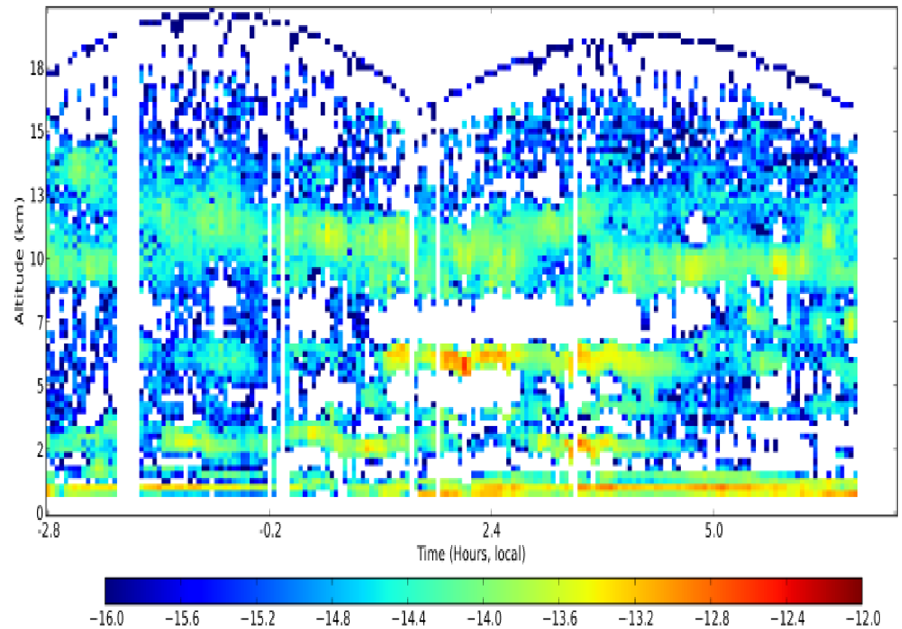
z zenith angle, λ the wavelength and $C_N^2(h)$ is the atmospheric turbulence strength at the altitude h.

r_0 is a function of the wavelength and increases as $\lambda^{6/5}$.

Atmospheric turbulence strength

$C_N^2(h)$ is the profile of the atmospheric strength as function of the altitude h.





Log C_N^2 function of time and altitude.

JKT observatory, La Palma - Canary Islands, May 2013

Seeing versus diffraction limit

r_0 is the equivalent diameter of a seeing limited scope of aperture D>>r_0

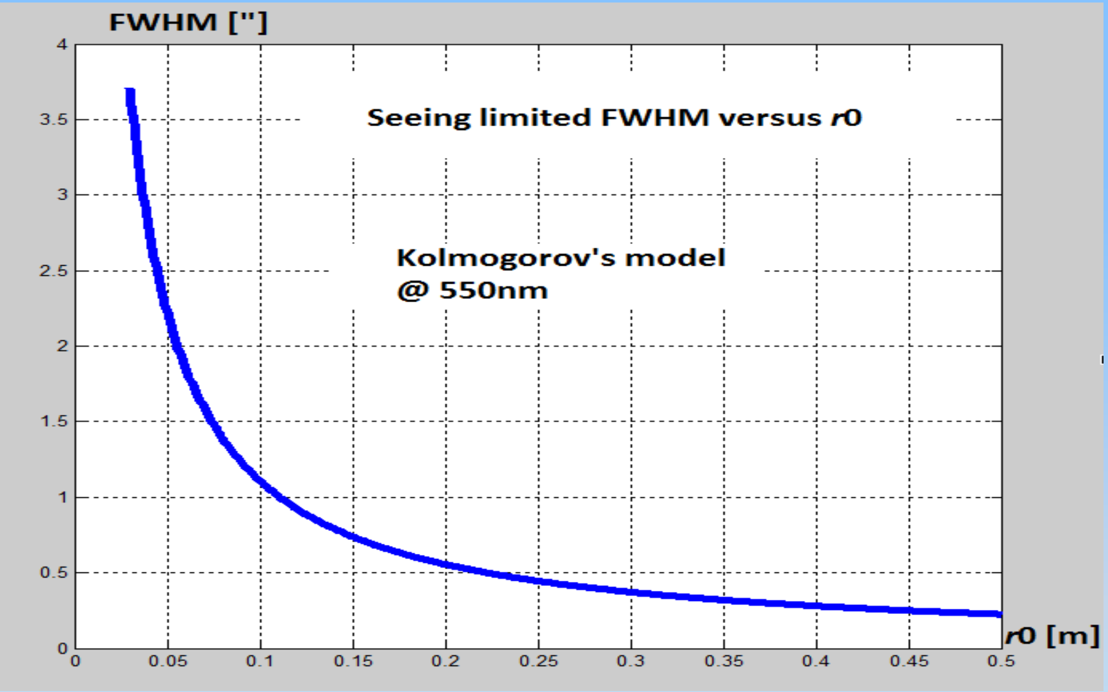
Diffraction limited $\frac{D}{r_0} < 1$

$$\mathbf{FWHM} \cong \frac{\lambda}{D} \ [\mathbf{radian}]$$

Seeing limited $\frac{D}{r_0} > 1$

$$\underbrace{\mathbf{FWHM}}_{long\ exposure} \cong \frac{\lambda}{r_0} \ [\mathbf{radian}]$$

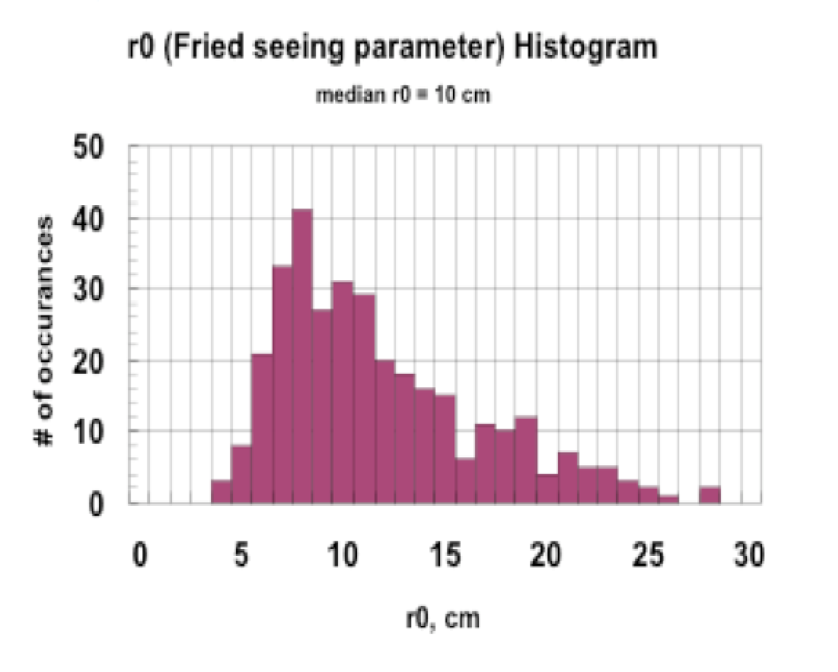
$$'' \cong \mathbf{radian} \times 2 \cdot 10^5$$

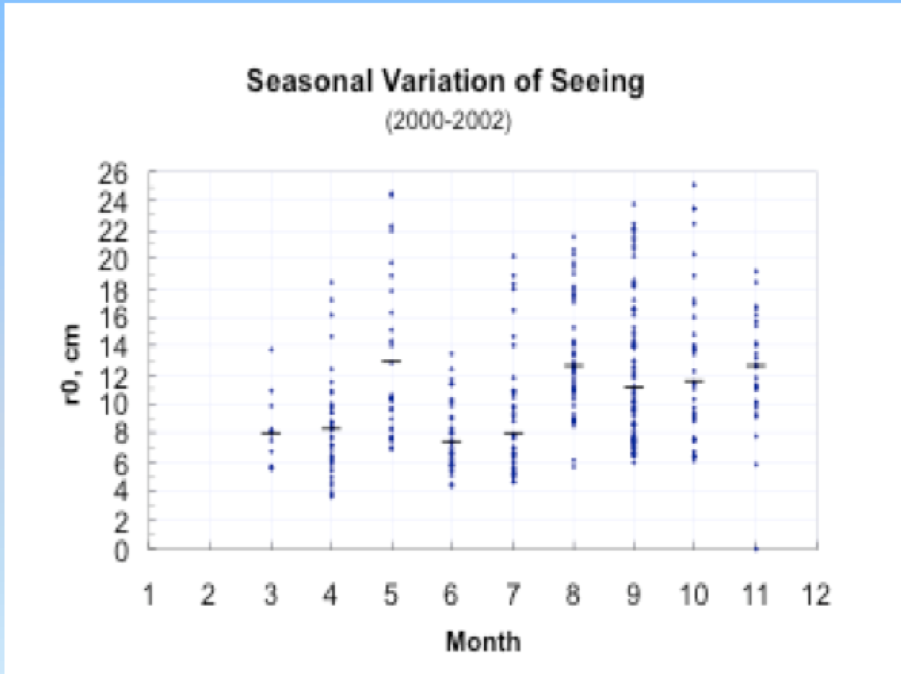


FWHM [“]	1	1.5	2	2.5	3
r_0 [mm/inch]	110 / 4.3	74 / 2.9	56 / 2.2	44 / 1.7	37 / 1.5

r_0 statistics

r_0 histogram and seasonal variations for Lick Observatory (CA USA).

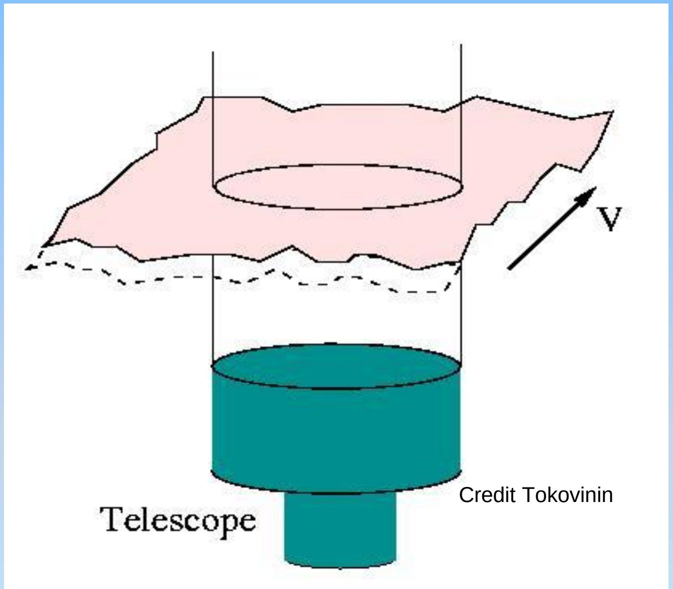




Coherence time τ_0

$\boldsymbol{\tau_0}$ corresponds to the time for which the changes in the turbulence becomes significant (1 radian rms, $\sim\lambda/6$). One assumes that the turbulence pattern is moved by the wind faster than it evolves:

$$\boldsymbol{\tau_0 \cong 0.31\frac{r_0}{v}}$$



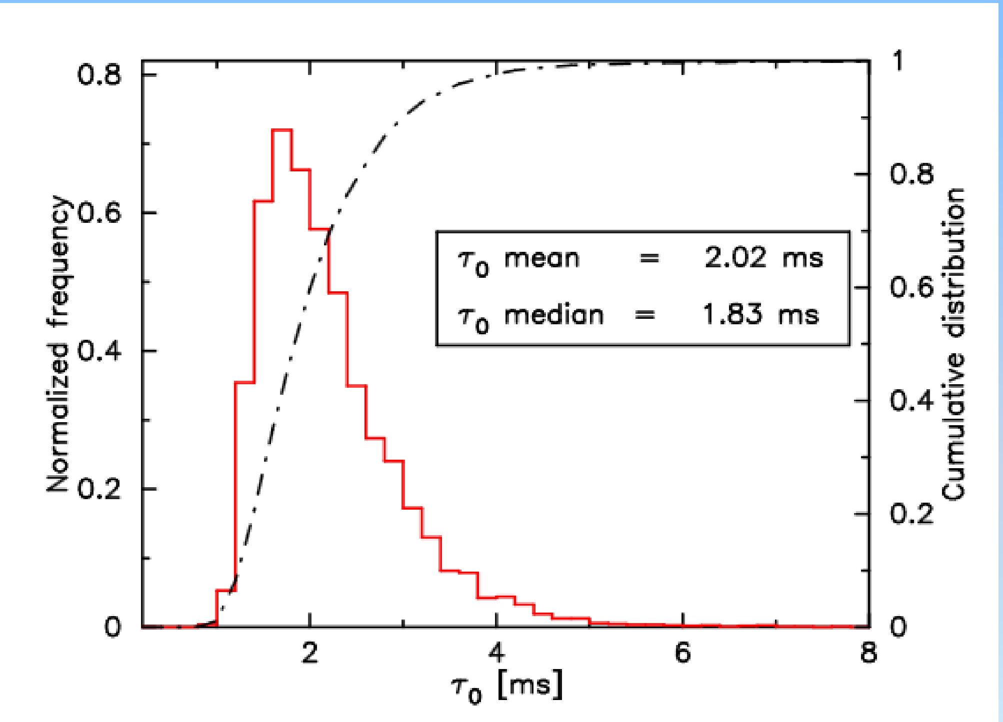
where $\boldsymbol{v}$ is the average wind speed
Typical $\boldsymbol{v}$ ~10m/s (~20mph)

$$\boldsymbol{r_0 = 50\text{mm} \;\rightarrow\; \tau_0 = 1.5\text{ms}}$$

τ_0 is a function of the wavelength and increases as $\lambda^{6/5}$.

τ_0 Statisitics

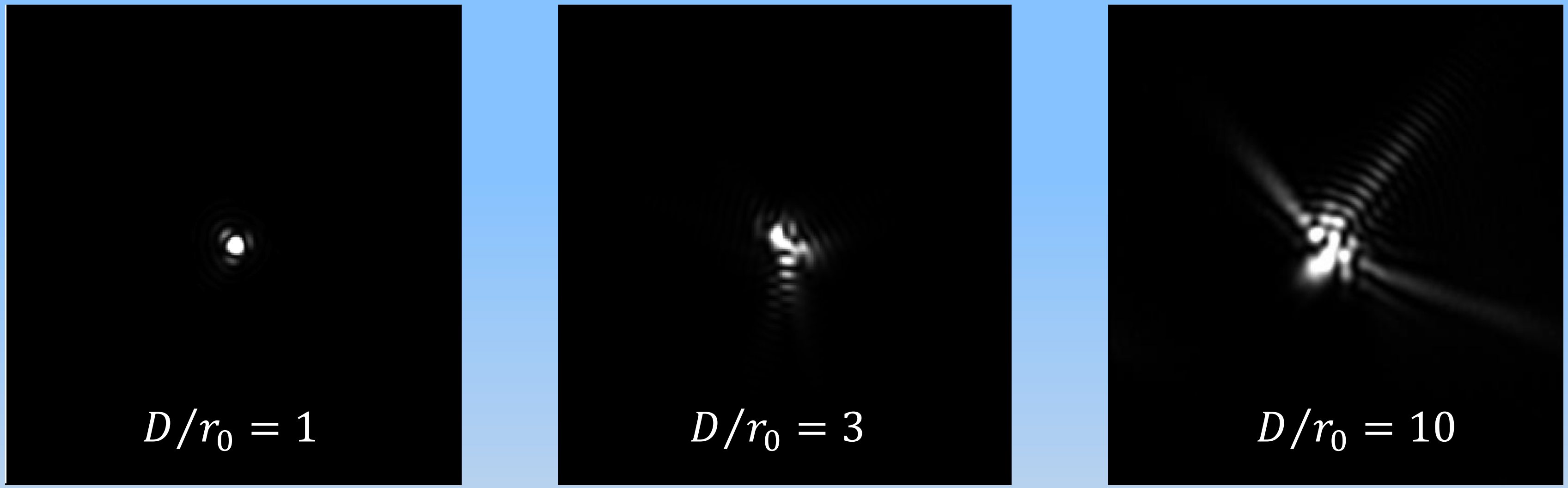
τ_0 histogram & cumulative distribution for Xinglong observatory (China).



r_0 [mm] @ 550nm	τ_0 [ms] V = 10m/s (~20mph)
30	0.93
50	1.5
70	2.2
100	3.1
300	9.3

Short term seeing t << τ_0

For exposure times t<<τ_0 the seeing is "frozen".



Nb. of speckle $\sim(D/r_0)^2$

Nb. of AO degree of freedom $\sim(D/r_0)$

D/r_0 decreases as $\lambda^{6/5}$

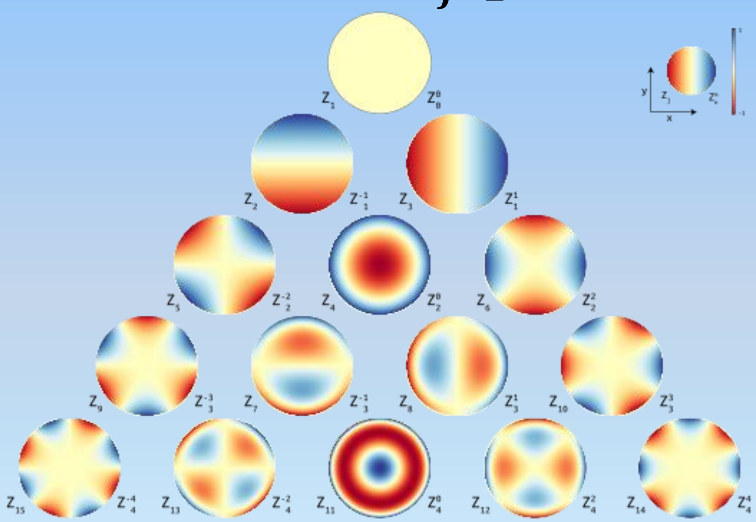
Diffraction limited operation is easier at longer wavelengths

Aberrations and seeing

Atmospheric energy transfer goes from the largest (L_0) down to smaller scales (l_0), the strongest distortions are on the lowest-order Zernike's modes (~tilt/tip, piston ignored).

$$\langle \phi^2 \rangle = \left(\frac{D}{r_0}\right)^{5/3} \sum_{j=2}^{\infty} \Delta_j$$ (Noll, 1976)

$\langle \phi^2 \rangle$ decreases as $\lambda^{1/2}$



Zernike's polynomials

Type of aberration removed (inclusive)	Residual rms phase error $\sqrt{\Delta}$ [%]
Tilt/tip	36%
Defocus	33%
Astigmatism	25%
Coma (3rd order)	23%
Spherical (4th order)	21%
Trefoild (3rd order)	19%

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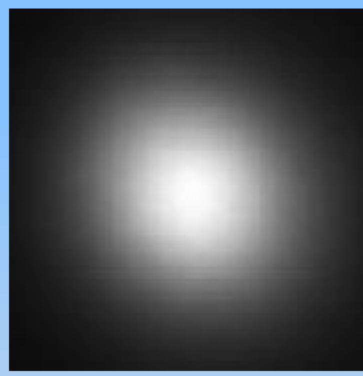
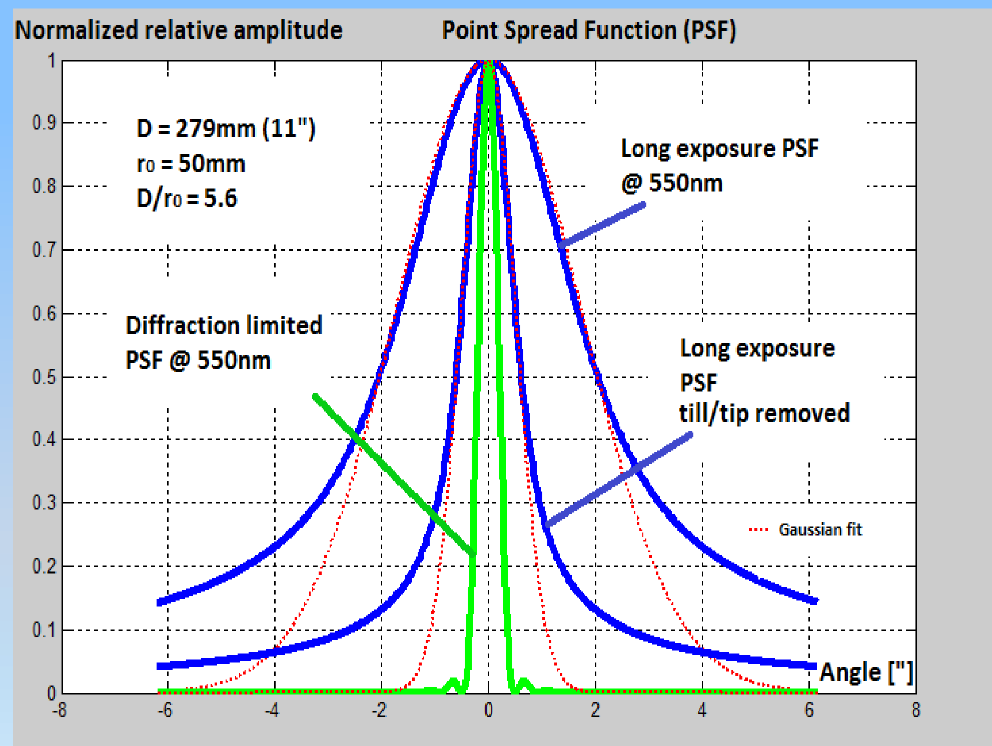
Long term seeing t >> τ_0

For exposure times t >> τ_0 the seeing is averaged.

$$\mathrm{FWHM} \cong \frac{\lambda}{r_0}$$



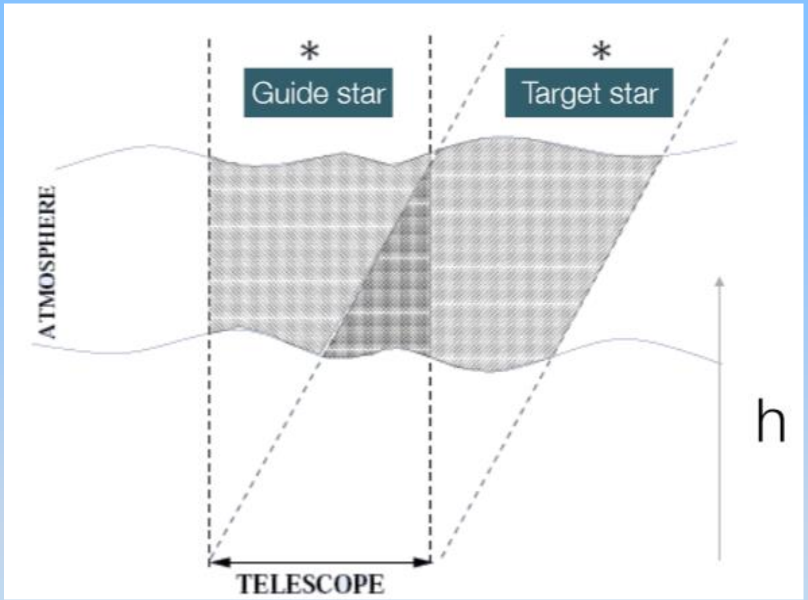
$$\mathrm{FWHM} \cong 0.3\frac{\lambda}{r_0}$$



Isoplanatic patch

The angle for which the wavefront error remains almost the same (~λ/6) is known as the isoplanatic angle:

$$\boldsymbol{\theta_0 \cong 0.31\frac{r_0}{h}}$$

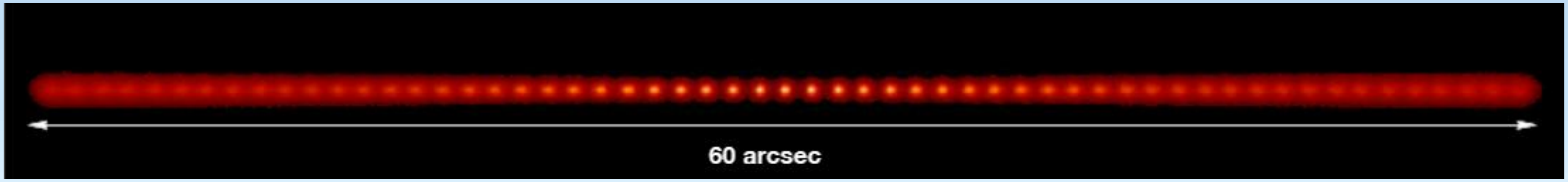


$\boldsymbol{h}$ ~ 5km, $\boldsymbol{\theta_0}$ is usually few arc-second across (@550nm):

$\boldsymbol{r_0}$ = 50mm → ~ 0.6”

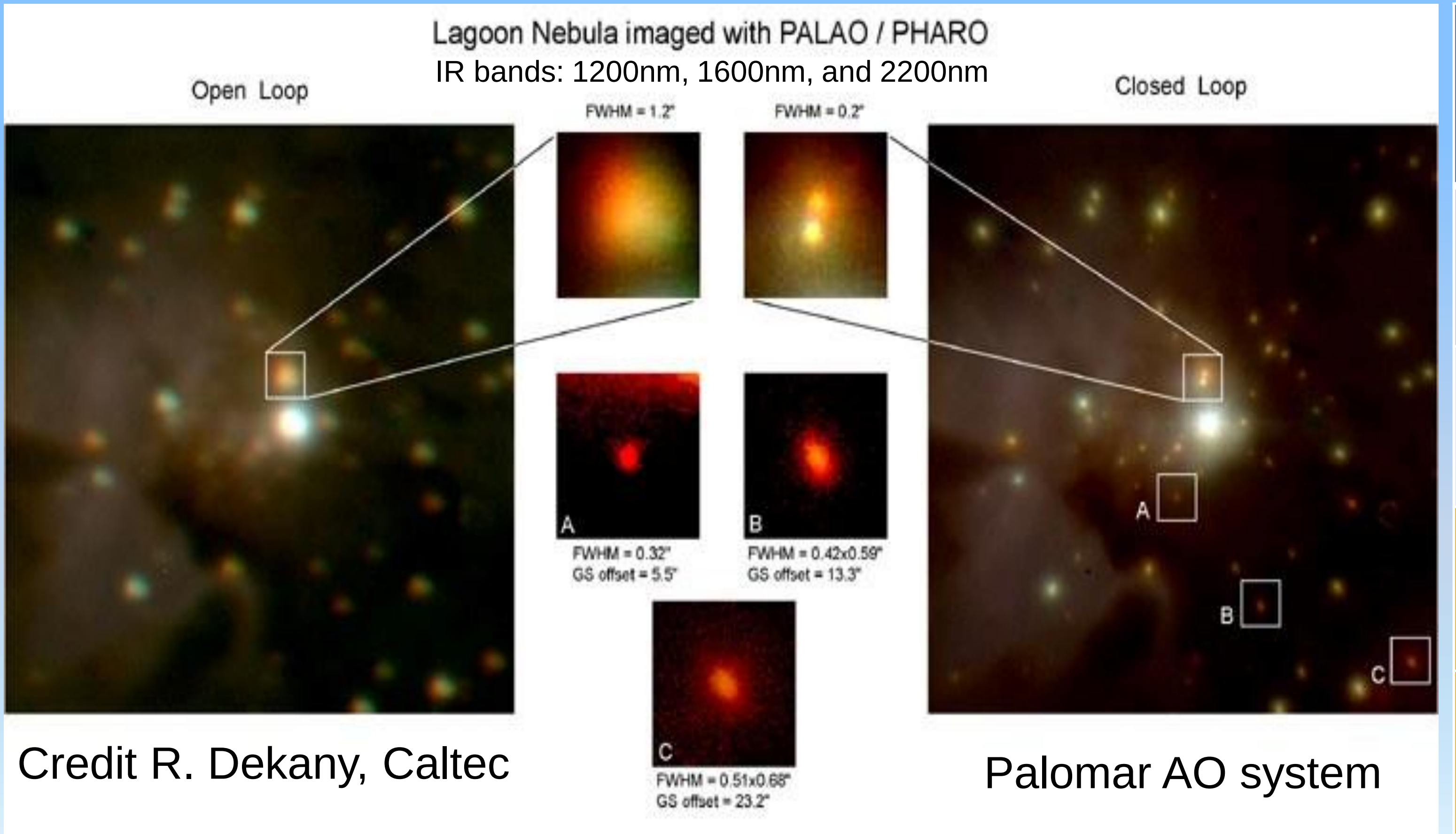
$\boldsymbol{r_0}$ = 200mm → ~ 2.6”

$\boldsymbol{\theta_0}$ increases as $\lambda^{6/5}$



Effect of the isoplanatic angle on AO

AO operation is usually only effective in a very narrow FOV.



Guide star offset ["]	FWHM ["]
0	0.2
5.5	0.3
13	0.45 - 0.59
23	0.51 - 0.68

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Isoplanatic angle and exposure time

Temporal phase fluctuations derive from spatial fluctuations under the “frozen turbulence” model. Turbulence pattern moves “untouched” at the average wind speed across the aperture.

Small cells contribute to the high frequency turbulences and are eventually averaged during image exposition (t_{exp}) at the expense of the resolution (larger star FWHM).

Large cells are related to low frequencies.
Long exposure times increase the apparent isoplanatic angle:

$$\boldsymbol{\theta_{exp} \cong \theta_0 \left(\frac{t_{exp}}{\tau_0}\right) = \frac{v}{h} t_{exp}} \qquad \mathrm{D} > \boldsymbol{r_0}, \quad \boldsymbol{t_{exp} > \tau_0}$$

Temporal behavior of seeing

$\boldsymbol{v}$ ~ 10m/s (~ 20mph), $\boldsymbol{h}$ ~ 5km, D = 279mm (11”) @550nm
Diffraction limit = 0.4”

<u>Tilt/tip corrections v.s. seeing function of exposure time</u>

t_{exp}	0	50ms	0.5s	1s	5s	10s
θ_{exp}	**0.6” 0.01’**	**20” 0.3’**	**200” 3’**	**400” 7’**	**2000” 33’**	**4000” 67’**
FWHM tilt/tip removed r_0=50mm	**0.7”**	**1.5”**	**2.2”**	**2.2”**	**2.3”**	**2.3”**
FWHM tilt/tip removed r_0=75mm	**0.5”**	**1.0”**	**1.5”**	**1.5”**	**1.5”**	**1.5”**

Lucky imaging and seeing

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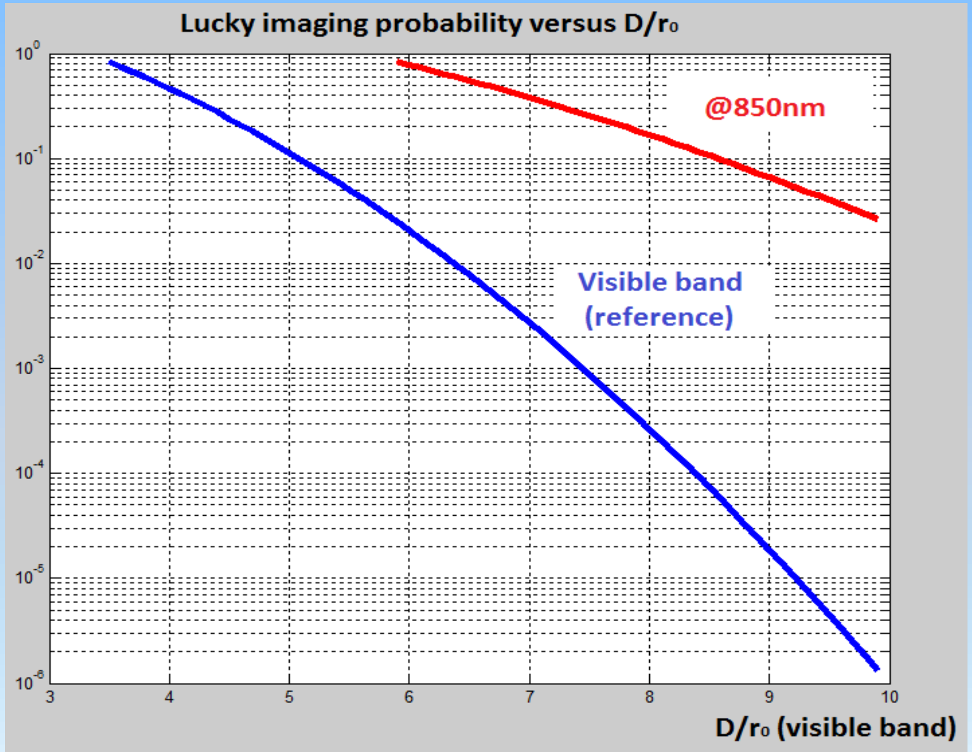
Some time a short exposure image can be close to the diffraction limit. The probability $\boldsymbol{P}$ for a rms phase error at, or below, ~λ/6 (one radian) is (Fried 1977):

$$\boldsymbol{P} \cong \mathbf{5.6}\, e^{-0.1557\left(\frac{D}{r_0}\right)^2}$$

$\frac{D}{r_0}$ >3.5, t<<τ_0, inside isoplanatic patch

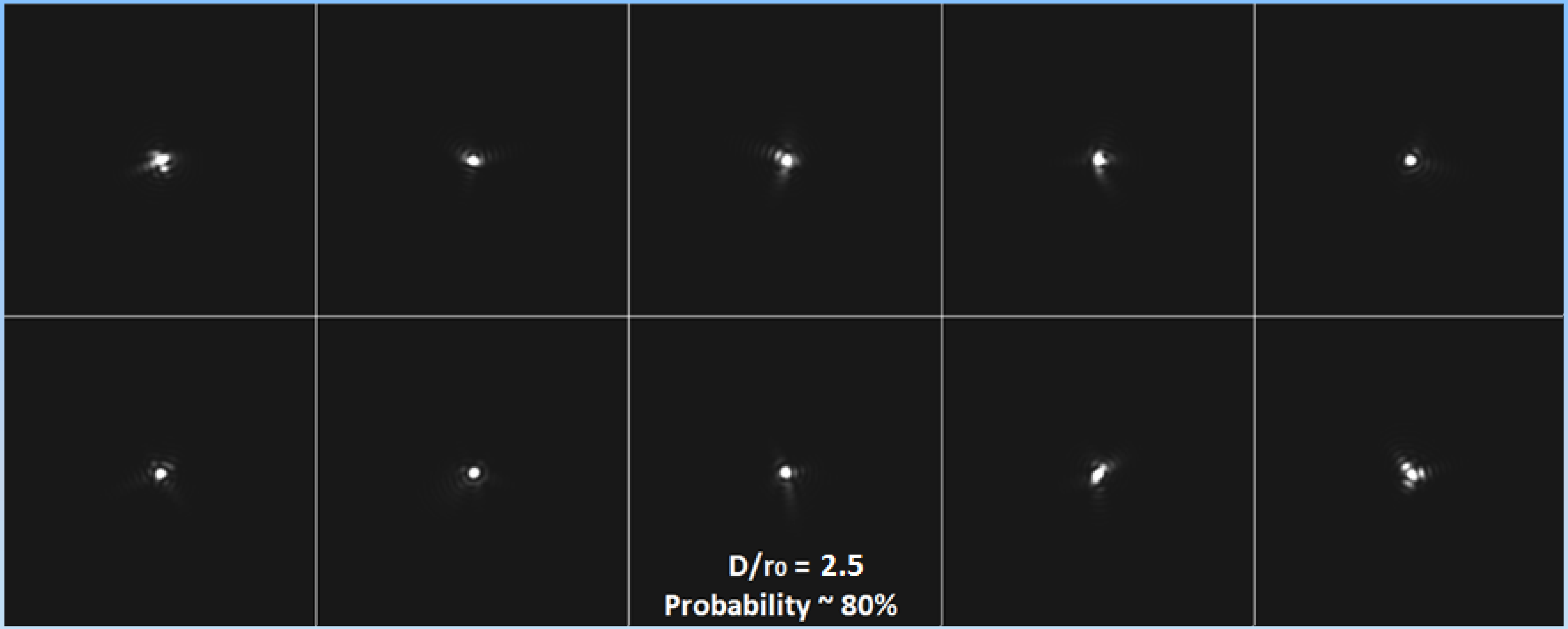
<u>Example:</u>

D=279mm (11")
r_0=50mm @550nm
$\boldsymbol{P}$ ≅0.04 (~1/25 frame)
r_0=84mm @850nm
$\boldsymbol{P}$>0.7 (~18/25 frame)



Lucky imaging and PSF shape

Set of 10 short exposure frames (tilt/tip removed):



Seeing and wavelength

The seeing is wavelength dependent. It is worse at shorter wavelengths and better at longer wavelengths.

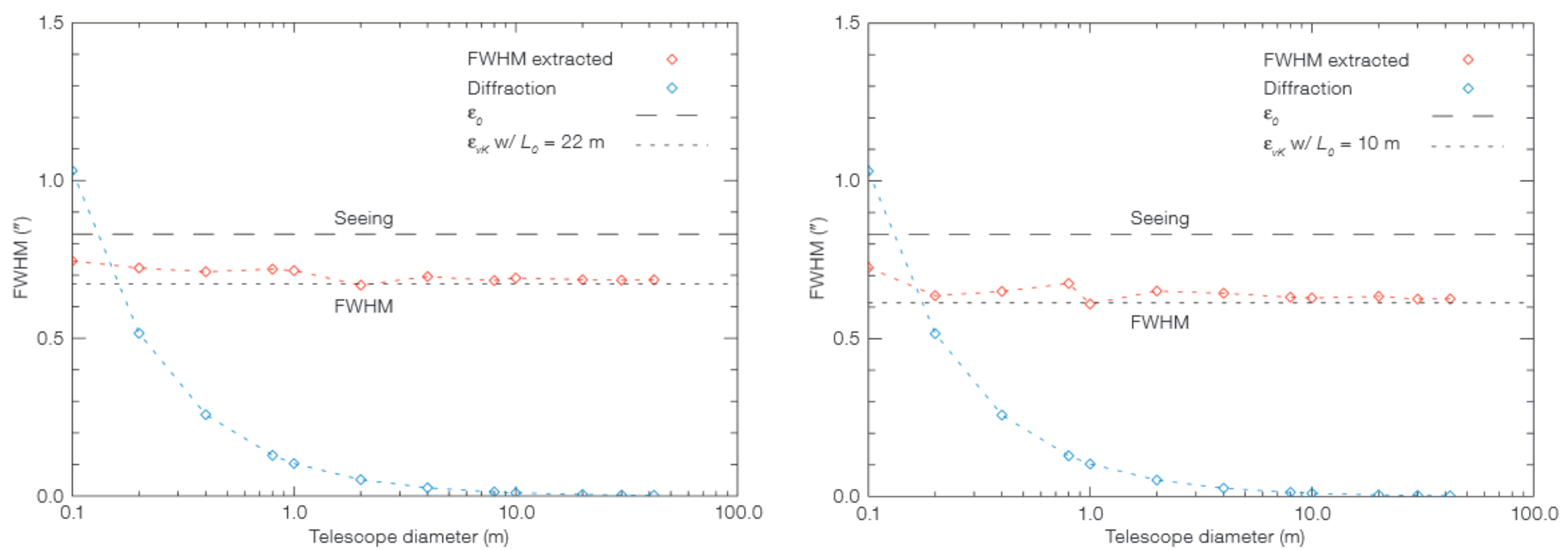
- $\boldsymbol{r_0(\lambda)} \quad \propto \boldsymbol{\lambda^{6/5} = \lambda^{1.2}}$ coherence length (Fried)
- $\boldsymbol{\tau_0(\lambda)} \quad \propto \boldsymbol{\lambda^{6/5} = \lambda^{1.2}}$ coherence time
- $\boldsymbol{\theta_0(\lambda)} \quad \propto \boldsymbol{\lambda^{6/5} = \lambda^{1.2}}$ isoplanatic angle (patch)
- $\boldsymbol{\langle \phi(\lambda)^2 \rangle} \quad \propto \boldsymbol{\lambda^{-5/3} \sim \lambda^{-1.7}}$ wave front phase variance
- $\mathbf{FWHM} \quad \propto \boldsymbol{\lambda^{-1/5} = \lambda^{-0.2}}$ Full Width at Half Maximum
- $\boldsymbol{D/r_{0(\lambda)}} \quad \propto \boldsymbol{\lambda^{-6/5} = \lambda^{-1.2}}$ DL performance threshold
- $\boldsymbol{P(\lambda)} \quad \propto \boldsymbol{\lambda^{12/5} = \lambda^{2.4}}$ Lucky imaging rate of success

Limitations of the standard Kolmogorov's model

- The Kolmogorov's model is only valid for $L_0 \to \infty$ and $l_0 \to 0$.
- When the outer scale turbulence (OST) L_0 is finite the seeing may exhibit a significant departure from the model.
- When the low level turbulence dominates L_0 is small, around 10m, possibly down to about 1m.
- The resulting effect is the greatest for the first two (tilt/tip) Zernike terms.
- The seeing usually exhibits a stronger dependency with λ compared to the standard Kolmogorov's model.

The Von Karman's (VK) model

- The Von Karman's (VK) model explicitly accounts for a finite OST L_0 value (Theodore Van Karman, 1948).
- Worldwide extensive measurements correlate with VK model.



Source/credit: On the difference between seeing and image quality: When the turbulence outer scale enters the game. ESO "The messager 141", September 2010

Patrice Martinez
Johann Kolb
Marc Sarazin
Andrei Tokovinin

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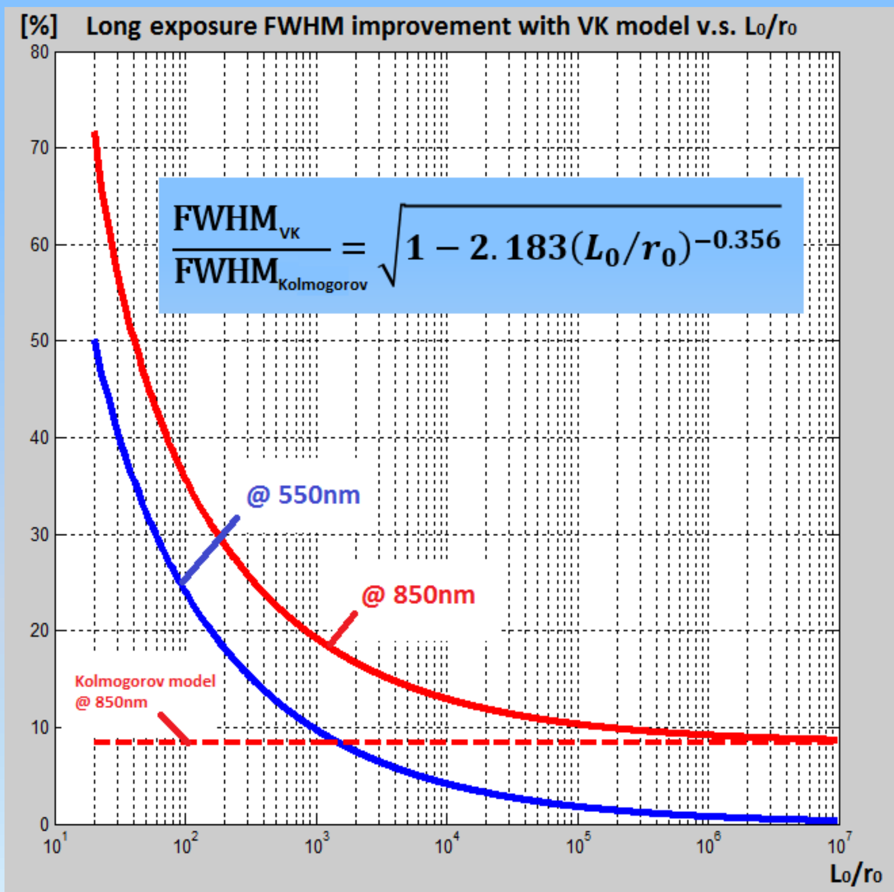
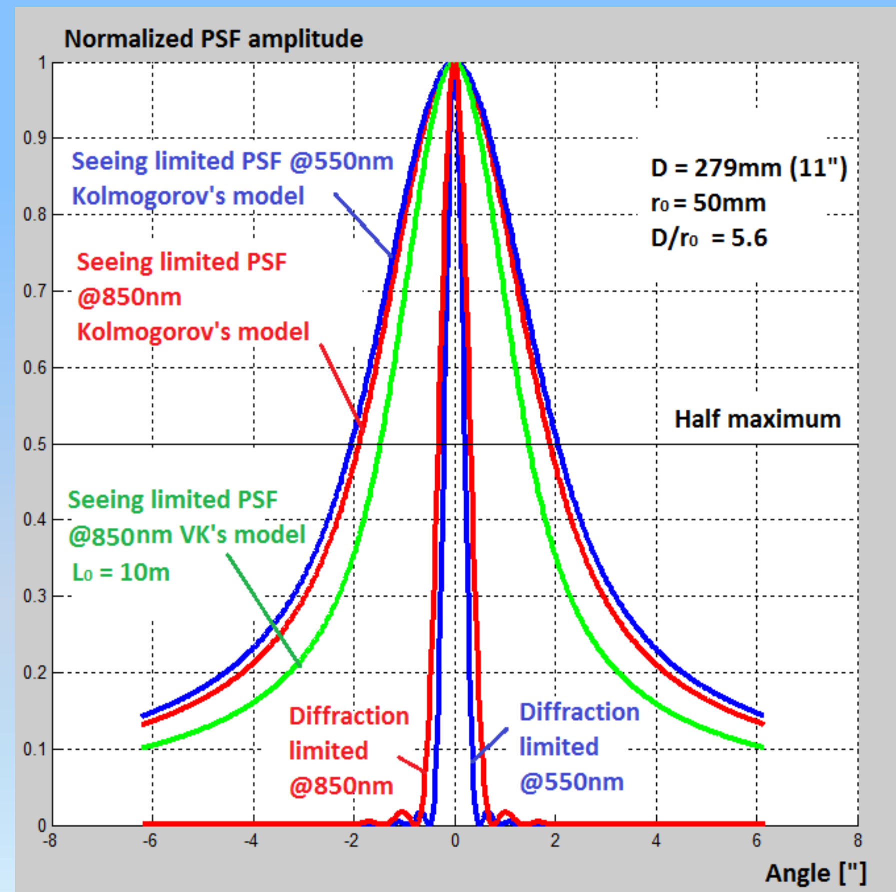
FWHM and the VK's model

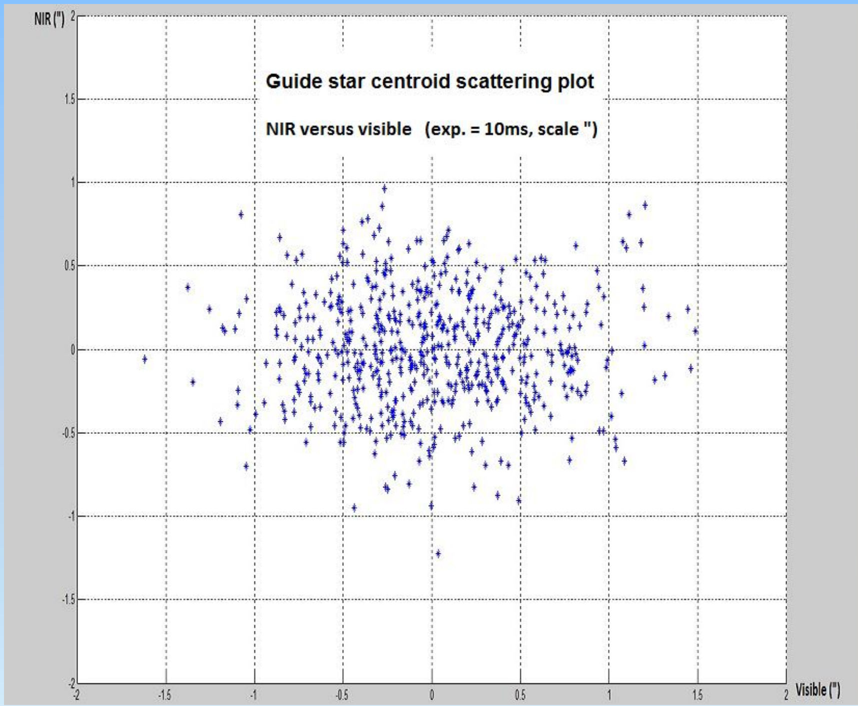
The VK model depends on a new parameter L_0/r_0

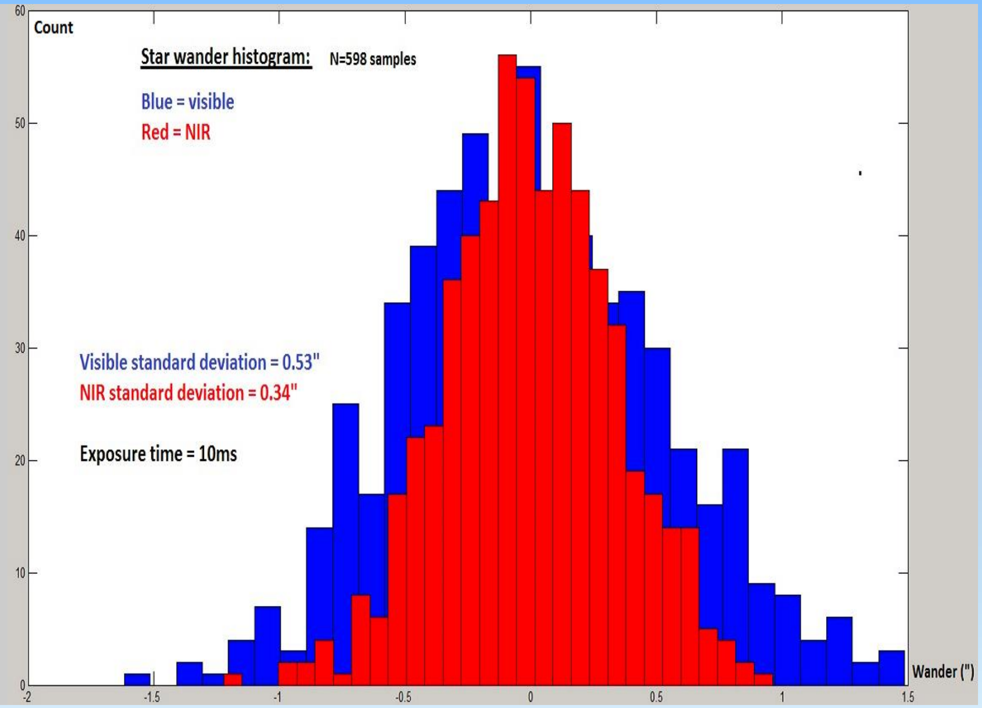




Finite OST and guide star wander

- Finite OST values improve the guide star PSF (less tilt/tip).
- Longer wavelengths benefice the most.





Auto-guiding

- The goal of auto-guiding is to correct for tracking errors.
- Those errors are independent of the seeing effect and are fully correlated across the FOV (unlike seeing).
- <u>Long term errors</u>: PE, King' rate, flexure, polar alignment, …, require a slow correction rate (10s to minutes).
- <u>Short term errors</u>: Mount mechanics, wind burst, vibrations, …, require a fast correction rate (few seconds).
- Short term corrections are more prone to seeing effects.

Auto-guiding and seeing

- The acceptable tracking error is a function of the seeing.

<u>*Rule of thumb:*</u> *RMS tracking error < 1/4 FWHM$_{seeing}$*

Seeing	Excellent 0.5”	Good 1.0”	Average 2.0 ”	Bad 3.0”
$\sigma_{tracking}$ rms error	0.13”	0.25”	0.50”	0.75”

- The rms auto-guiding error $\sigma(t_c)_{guiding}$, before any correction, for a given exposure time t_c is given by:

$$\sigma(t_c)_{guiding} = \sqrt{\sigma(t_c)^2_{tracking} + \sigma(t_c)^2_{seeing}}$$

Auto-guiding rates

- Let's assume that t_c has been chosen to cancel the tracking errors (mount, setup).

- <u>Question:</u> How does the seeing impact auto-guiding errors?

- <u>Answer:</u> It depends whether we are inside or outside the isoplanatic patch.

- Total error after correction **<u>inside</u>** the isoplanatic patch:

$$\boldsymbol{\sigma}(t_c)_{\boldsymbol{guiding}} \cong (\mathbf{1}-\boldsymbol{K})\,\boldsymbol{\sigma}_{\boldsymbol{seeing}} \qquad \mathbf{0} \le \boldsymbol{K} \le \mathbf{1}$$

$\boldsymbol{K} \to \mathbf{0}$ seeing limited image: $\boldsymbol{t_c} \gg \boldsymbol{\tau_0}$

$\boldsymbol{K} \to \mathbf{1}$ ~ diffraction limited image: $\boldsymbol{t_c} \ll \boldsymbol{\tau_0}$ (tilt/tip correction only)

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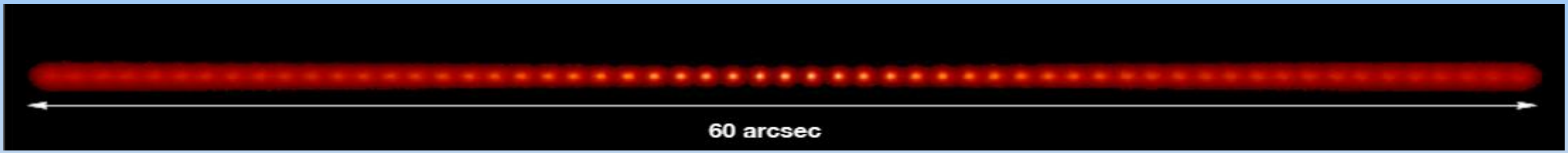
Auto-guiding rates (cont.)

- Total error after correction **outside** the isoplanatic patch:

$$\boldsymbol{\sigma}(t_c)_{guiding} \cong \left(\sqrt{\mathbf{1} + \boldsymbol{K}^2}\right) \boldsymbol{\sigma}_{seeing} \qquad \mathbf{0} \leq \boldsymbol{K} \leq \mathbf{1}$$

$\boldsymbol{K} \rightarrow \mathbf{0}$ seeing limited image: $\boldsymbol{t_c} \gg \boldsymbol{\tau_0}$

$\boldsymbol{K} \rightarrow 1$ ~**1.4 x seeing limited image**: $\boldsymbol{t_c} \ll \boldsymbol{\tau_0}$



- ***K*** should be kept as close as possible to zero.

$\boldsymbol{t_c} \gg \boldsymbol{\tau_0}$, guiding at longer wavelengths, better mount

Astronomy

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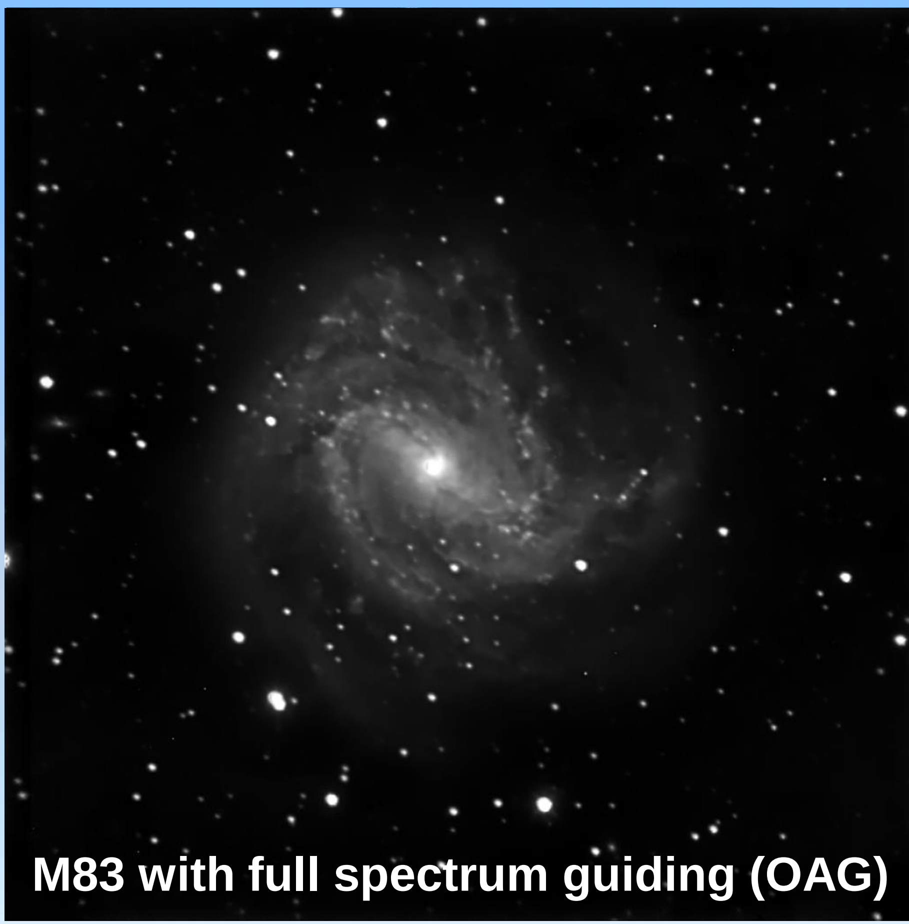
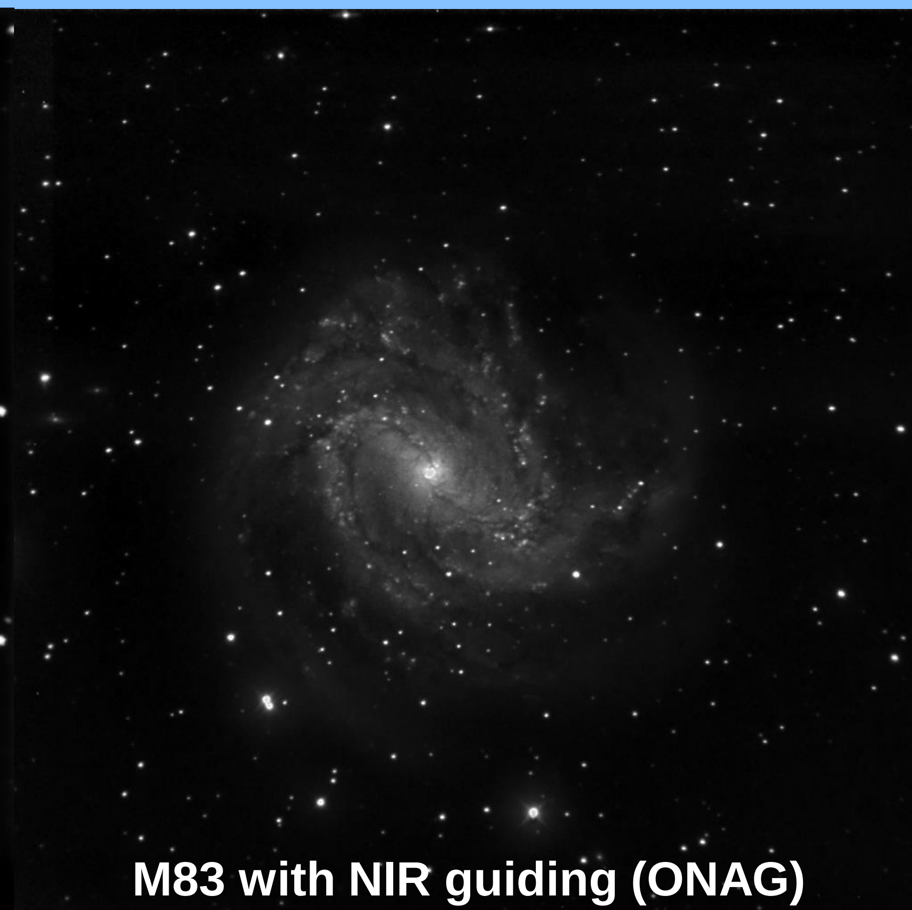
NIR versus visible auto-guiding

Mario Motta's relay telescope 32" (0.8m) @ f/6
M86 images with STL11000 + AO-L



M83 with full spectrum guiding (OAG)



M83 with NIR guiding (ONAG)

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Some advices

- **Do not rush auto-guiding**, unless you have too (>5~10s).
- Use a **long guiding exposure time** whenever possible.
- Minimize errors with a **good polar alignment**.
- **Use a model** for minimizing the drifts and the guiding rate.
- Consider **guiding at longer wavelengths** (such as NIR).
- **Do not minimize the guiding error** at any cost.
- Use **small aggressiveness** at fast guiding rates.
- Consider using **red filters** for image detail retrieval.
- Image **high above the horizon** to minimize seeing.
- Know your **local seeing and conditions**.
- **Watch forecasts** (cleardarksky.com) to mange expectations.

Thank you!

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Clear skies!